

THE NEW ART OF
LAND MEASURING;
OR, A
TURNPIKE ROAD to
Practical SURVEYING:

Leading to a new and exact Method of MEASURING and
MAPING of LANDS, WOODS, WATERS, &c. by the
CATOPTRIC SEXTANT, and to cast up
the same by the PEN only.

A L S O,

To many new Discoveries in LAYING OUT, DIVIDING,
and REDUCING of LAND.

To Levelling for the Conveyance of WATER, either
in PIPES or open CANALS:

TOGETHER WITH AN

A P P E N D I X,

CONTAINING A

New THEORY of the CATOPTRIC SEXTANT,

AND ITS FARTHER USE,

In an entire new METHOD of taking HEIGHTS and DISTANCES,
independent of TRIGONOMETRY:

A L S O,

MEASURING of STANDING TIMBER. To which are added,
Several new and useful TABLES.

The Whole illustrated with COPPER-PLATES.

BY B. TALBOT,
Teacher of the MATHEMATICS at CANNOCK.

WOLVERHAMPTON:

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THE HISTORY OF THE
ARTS AND MANUFACTURES
OF GREAT BRITAIN
AND IRELAND
FROM THE EARLIEST
TIMES TO THE PRESENT
PERIOD
BY
JOHN HENRY COLEMAN
OF THE
SCHOOL OF MINES
LONDON
PRINTED BY
JOHN JOHNSON
ST. PAULS CHURCH-YARD
1837

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P

T H E
P R E F A C E.

AS a great number of books of Surveying have already been published, and some of them by very eminent authors, it is therefore necessary that I should give my reasons for adding one more to the number. Having in my youthful days practised Surveying by the Plain Table ; in which I found many inconveniences and errors, such as the often shifting the paper, unless a scale too small for determining the contents, to any tolerable degree of exactness, was made use of ; likewise the paper's expanding in wet or
a damp

damp weather, in the field where the dimensions were taken ; and contracting in the house in a warm room where the contents are generally cast up. These, and some other inconveniences I met with, induced me to lay this instrument aside, and try the Theodolite. The Theodolite I made use of, was of my own constructing, and made by that very ingenious mechanic Mr. *John Baddeley* ; it was eleven inches diameter, having a vertical arch, telescopic sights, and two spirit levels, and moved by tooth and pinion, adjusting screws, &c. Being thus equipped with a grand theodolite, I practiced Surveying according to the methods laid down by *Wild, Hammond, &c.* in their books of Surveying, but I soon found this instrument too cumbersome and heavy to be conveniently carried about, and also liable to
be

be damaged if trusted in the hands of persons unacquainted with its use. And having some time after constructed a large *Hadley's Octant* of thirty inches radius, (for measuring the moon's distance from the sun and stars, in order for finding the longitude at sea, and in which I have fully succeeded, but cannot yet get it introduced to the *Board of Longitude*, though far preferable to any they have published), by which I soon found I could measure angles in all positions, as well horizontal as vertical, as large as 90 degrees; and began to think that an instrument of this sort may be of use in surveying of land, especially if it could be so constructed as to measure at once an angle greater than 90 degrees; and accordingly I constructed and made one of fifteen inches radius, whose arch or limb was the sixth

part of a circle, and with which I could measure an angle of 120 degrees; this I found was in general, sufficient to measure all the four angles of a trapezium: it was quite portable, and I could take an angle in one fifth of the time required to do the same by the theodolite, and therefore thought it a very proper substitute for that instrument. Being still unsatisfied with the common methods of planning for the contents, and of the multiplicity of angles and station lines to be measured, and then to be plotted down; and that they seldom or never would close, and consequently the contents could seldom be right. I therefore contrived to measure all pieces or parcels of land, by three or four station lines, and three or four angles only; for considering that the two sides of a triangle being multiplied together, and that pro-
duct

THE PREFACE.

duct multiplied by the natural sine of the included angle, this last product would be double the area of the triangle; and whereas a trapezium contains two triangles by drawing a diagonal, its content might be found in like manner, at or by two operations. From this consideration I constructed table the 3d of angles and factors (see page 95), being half the natural sines of the angles they stand against, reduced to the radius 1. Having discovered this most accurate method of surveying; I advertised in ARIS's *Birmingham Gazette*, in 1763, to teach this my new method of Land Measuring by the Catoptric Sextant, whereby all lands were cast up by the pen only. And this sextant, and this method, I believe were first invented, and first made use of in surveying by me; as I had neither heard of nor seen any such before,

notwithstanding the catoptric sextant is now become a very common and useful instrument.

Having not only taught, but practiced this new method, now about sixteen years, and can pronounce it to be the most expeditious, the most easy and the most exact method yet known, or at least yet published; as it not only admits of the land being cast up by the pen only, without having any regard to scale and compasses; but also admits of a certain proof whether the dimensions are taken right or not: and therefore thought I could not do a more beneficent act than make it public.

Perhaps some of my readers may surmise that I am unacquainted with the method of surveying by (what some surveyors call) the long line, and in which
the

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the chain only is used ; this method is not only very laborious, but as in the method by the plain table and theodolite, depends upon the scale and compasses for the contents.

In the following work will be found many other things entirely new, and not to be met with in any other author ; of such is the 22nd Prob. of Geometry, shewing now to divide a quadrant, or any arch of a circle, into any number of equal parts, as far as the fourth part of a degree, or fifteen minutes, by continual bisection ; being of use in dividing of mathematical instruments.

Likewise a small table for finding the areas of the segments of circles, (without the trouble of finding the circle's diameter) by having the chord and height of the segment given ; being of great use in

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finding the areas of off-sets, when the fences or part of them are curve lines; which are very common in old inclosures. And though this table contains but eleven factors, it is sufficient for the use of the surveyor, and may be further extended for the nicer parts of mensuration, such as gauging, &c.

In the division of land are several new problems particularly the 2d. 8th. and 9th. ; the 2d. shews how to divide land, such as commons, &c. according to quantity and quality. The 8th. shews how to divide any right lined figure in any proposed ratio, by right lines from a point in one of it's sides, by pure geometry. The 9th. shews how to do the same from a point near the middle.

In the Appendix, I have given an entire new method of measuring heights and distances,

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distances, both accessible and inaccessible; and that without having the least knowledge of trigonometry; and also of measuring standing timber, nearly as exact and expeditious as if fell.

I have also added a new table for the valuation of land.

Many other things of consequence will be found in this work, as may be seen in the table of contents; but are too numerous to be specified in a preface.

I am afraid that some of my readers will complain that I have omitted County and Coast Surveying; and indeed, as it might have been easily added in the manner it is done by other authors, containing only a few pages, may give them some pretence. But I can assure them, that the many improvements I have made, and that are absolutely necessary for making a
compleat

compleat county map, would have swelled this volume too much. Besides, as any ingenious person well acquainted with what I have wrote in this book, may make a map of a county, as correct as from any other author.

I have also omitted the method of perspective published in those books which treat of the new improved theodolites; not but it may be done as well with the sextant as the theodolite, but because I think such as would be compleat drawers in perspective, had better learn the true geometrical principles of perspective.

However, as some of my readers may not have an opportunity of learning the above principles, and yet would like sometimes to take a perspective plan of a gentleman's seat, landscape, &c. I shall endeavour to shew them how it may be done,

done, though they are entirely unacquainted with the rules of perspective.

They must procure a staff about their own height, one end must be made sharp to stick in the ground, and the other must have a slit to stick in a small square piece of glass; upon this glass must be drawn with a diamond (or for want of that, with a fine nibbed pen, or a drawing pen with ink), fine parallel lines, both ways across the glass at equal distances, and at right angles, so shall there be formed a parcel of little squares; let these be figured as taught in the reducing of land by similar squares: next must be procured a piece of wire about a foot long, and bent at two or three inches from one end in a right angle, the other part remaining straight; to the part thus bent up, must be fixed a small circular piece of pasteboard or card, about

about two inches diameter, which may easily be done, by leaving a small kind of stile or handle to the pasteboard or card, by which it may be tied to the wire ; in the center of this pasteboard must be a small round hole to look through when in use ; this wire must slide very tight through a hole near the top of the staff, and at right angles to the plane of the glass, so that the planes of the pasteboard and glass may be nearly parallel, and the hole in the pasteboard to face nearly the middle of the glass ; and thus have they a most simple but compleat instrument for drawing in perspective.

When a plan is wanted, they must fix a clean piece of paper on a little (drawing) board, a little larger than they design their plan should be ; upon this paper must be drawn little squares like those on the glass,
only

only larger if the plan is wanted larger than the glass, and smaller if it is wanted less; these lines may be drawn with red ink, in order to distinguish the pencil lines the better while drawing; then, having fixed down the staff and adjusted the pasteboard, so that they may see through the hole, the whole of the object or objects (to be drawn) on the glass; then by observing through the hole in the pasteboard on what square, or part of a square, any part of the object falls on the glass; and marking the same on the corresponding square on the drawing board; if it be a building, first mark the out corners, then join them with right lines, (which may be easily done by help of a small ruler) then mark the windows, doors, chimnies, &c. all which being properly drawn and shaded as they appear to the eye on the glass, will give a
most

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most exact perspective plan ; and that in much less time than even the angles could have been taken with the best theodolite.

As this is likewise an invention of my own, I shall call it designing by similar squares, or mechanic perspective.

This method of designing is applicable to all sorts of objects, as well within doors as without, and therefore may be said to be universal, and consequently universally useful.

A more grand instrument indeed may easily be constructed, but, perhaps, not a more useful one.

I have only to add, that I have endeavoured to make this work, as perfect, and as useful, as possible ; as to full perfection, it must not be expected from human nature ; (it is only the works of a Supreme Being that are absolutely perfect ;) but it
may

may be absolutely useful : and if it meets with such a character from my friends and the public, I have my wishes.

As to the critics, I hope they will have candour enough to pass over a few tautologies, misconstructions, or slips of grammar, as I was not studying grammar, but geometry. Upon the whole, if I have added any thing useful to mankind, I shall think I have not spent my vacant hours in vain, as we all ought to be useful members one to another.



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- 25. line 12, for A n read AB.
- 32. line 2, for ABCDE read ABCDEFG.
line 10, for draw EE read draw EI.
- 67. last line but two, for to C read to F.
- 96. last line but three, second column, for 104₄ read 104₅.
- 151. second line of table under Fact. for ,69796 read ,66796.
in last line of ditto, for 7852 read 7859.
- 157. line 2, for 3° read 4°.
- 167. line 16, for 4,884 read ,4884.
- 181. line 9, for BCD read BAD.
- 230. last line but one, for 95,3329 read 91,468, and correct the sum in the following page accordingly
- 231. line 9th, for D A and A E read D A, A E and B F.
- 259. line 11, for and the read when the side.
- 275. line last but one, for to side read to the side.
- 277. line 5, for to any side read in any side.

EXPLANATION of the CHARACTERS made use of in this WORK.

= equal; + plus, or more; — minus, or less;
 × multiplied; $\frac{6432}{543}$ division, or the upper figures
 divided by the under ones; 2 : 4 :: 6 : 12 propor-
 tion; √ square root; ∠ angle.



A TABLE for the VALUING of LAND.

Sh.	3 r.	2 r.	1 r.	30 p.	20 p.	10 p.	9 p.	8 p.	7 p.	6 p.	5 p.	4 p.	3 p.	2 p.	1 p.
	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>
1	0 9	0 6	0 3	0 2 $\frac{1}{4}$	0 1 $\frac{1}{2}$	0 0 $\frac{3}{4}$	0 0 $\frac{3}{4}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{4}$	0 0 $\frac{1}{4}$	0 0 $\frac{1}{4}$	0 0	0 0
2	1 6	1 0	0 6	0 4 $\frac{1}{2}$	0 3	0 1 $\frac{1}{2}$	0 1 $\frac{1}{4}$	0 1 $\frac{1}{4}$	0 1	0 1	0 0 $\frac{3}{4}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{4}$	0 0 $\frac{1}{4}$
3	2 3	1 6	0 9	0 6 $\frac{3}{4}$	0 4 $\frac{1}{2}$	0 2 $\frac{1}{4}$	0 2	0 1 $\frac{3}{4}$	0 1 $\frac{1}{2}$	0 1 $\frac{1}{2}$	0 1 $\frac{1}{4}$	0 1	0 0 $\frac{3}{4}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{4}$
4	3 0	2 0	1 0	0 9	0 6	0 3	0 2 $\frac{3}{4}$	0 2 $\frac{1}{2}$	0 2	0 1 $\frac{3}{4}$	0 1 $\frac{1}{2}$	0 1 $\frac{1}{4}$	0 0 $\frac{3}{4}$	0 0 $\frac{1}{2}$	0 0 $\frac{1}{4}$
5	3 9	2 6	1 3	0 11 $\frac{1}{4}$	0 7 $\frac{1}{2}$	0 3 $\frac{3}{4}$	0 3 $\frac{1}{2}$	0 3	0 2 $\frac{1}{2}$	0 2 $\frac{1}{4}$	0 1 $\frac{3}{4}$	0 1 $\frac{1}{2}$	0 1	0 0 $\frac{3}{4}$	0 0 $\frac{1}{4}$
6	4 6	3 0	1 6	1 1 $\frac{1}{2}$	0 9	0 4 $\frac{1}{2}$	0 4	0 3 $\frac{1}{2}$	0 3	0 2 $\frac{3}{4}$	0 2 $\frac{1}{4}$	0 1 $\frac{3}{4}$	0 1 $\frac{1}{4}$	0 1	0 0 $\frac{1}{2}$
7	5 3	3 6	1 9	1 3 $\frac{3}{4}$	0 10 $\frac{1}{2}$	0 5 $\frac{1}{4}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	0 3 $\frac{1}{2}$	0 3	0 2 $\frac{1}{2}$	0 2	0 1 $\frac{1}{2}$	0 1	0 0 $\frac{1}{2}$
8	6 0	4 0	2 0	1 6	1 0	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	0 3 $\frac{1}{2}$	0 3	0 2 $\frac{1}{2}$	0 1 $\frac{3}{4}$	0 1 $\frac{1}{4}$	0 0 $\frac{1}{2}$
9	6 9	4 6	2 3	1 8 $\frac{1}{4}$	1 1 $\frac{1}{2}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4	0 3 $\frac{1}{2}$	0 2 $\frac{3}{4}$	0 2	0 1 $\frac{1}{4}$	0 0 $\frac{1}{2}$
10	7 6	5 0	2 6	1 10 $\frac{1}{2}$	1 3	0 7 $\frac{1}{2}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{4}$	0 4 $\frac{1}{2}$	0 3 $\frac{3}{4}$	0 3	0 2 $\frac{1}{4}$	0 1 $\frac{1}{2}$	0 0 $\frac{3}{4}$
20	15 0	10 0	5 0	3 9	2 6	1 3	1 1 $\frac{1}{2}$	1 0	0 10 $\frac{1}{2}$	0 9	0 7 $\frac{1}{2}$	0 6	0 4 $\frac{1}{2}$	0 3	0 1 $\frac{1}{2}$
30	22 6	15 0	7 6	5 7 $\frac{1}{2}$	3 9	1 10 $\frac{1}{2}$	1 8 $\frac{1}{4}$	1 6	1 3 $\frac{3}{4}$	1 1 $\frac{1}{2}$	0 11 $\frac{1}{4}$	0 9	0 6 $\frac{3}{4}$	0 4 $\frac{1}{2}$	0 2 $\frac{1}{4}$
40	30 0	20 0	10 0	7 6	5 0	2 6	2 3	2 0	1 9	1 6	1 3	1 0	0 9	0 6	0 3
50	37 6	25 0	12 6	9 4 $\frac{1}{2}$	6 3	3 1 $\frac{1}{2}$	2 9 $\frac{3}{4}$	2 6	2 2 $\frac{1}{4}$	1 10 $\frac{1}{2}$	1 6 $\frac{3}{4}$	1 3	0 11 $\frac{1}{4}$	0 7 $\frac{1}{2}$	0 3 $\frac{1}{4}$

The NEW ART of
LAND MEASURING.

SECTION. I.

Containing some useful Geometrical definitions.

GEOMETRY Originally signifies the art of measuring the Earth, or any distances or dimensions on or within it. But it is now used for the science of quantity, extension or magnitude, abstractly considered, without any regard to matter.

Geometry very probably had it's first rise in Egypt, where the Nile annually overflowing the country, and covering it

B

with

with mud; obliged men to distinguish their lands one from another, by the consideration of their figure; and to be able also to measure the quantity of it, and to know how to plot it, and to lay it out again in it's just dimensions, figure and proportion: After which it is likely a farther contemplation of those draughts and figures, helped them to discover many excellent and wonderful properties belonging to them; which speculation continually was improving, and is at this very day.

GEOMETRICAL DEFINITIONS.

I. A Point is the beginning of magnitude, and the bound of a line, and may be conceived to be infinitely small; though we may represent it by a small speck or dot with a pen thus (.)



II. A

II. A Line is generated or produced by the motion of a point; and is the bound of a superficies, and may be represented by the motion of a pen by the edge of a ruler, as from A to B. (Fig. 1.) It is either right when it is the shortest distance between the two points A and B; or curved, when it is not the shortest distance between two points as the line CD.

III. A Superficies is generated by the motion of a line, and has two dimensions, viz. length and breadth. A superficies may be contained within one curve line, but of right lines it cannot be contained in less than three.

N. B. Most authors and especially EUCLID, define a point to have no parts; or that it cannot be divided, no not even in thought. Yet a line is produced by

the motion of a point, and is said to have length, but neither breadth nor thickness. But how can that which has no parts produce magnitude or length? or how can nothing produce something?

IV. A Circle is a plain figure bounded by one curve line ABGD, called the circumference; the middle point whereof C, is called the center, from which all right lines as CA, CB, CF, CG, are equal, and called the radius. The diameter is a right line which passeth through the center and touches the circumference in two points as AG, (Fig. 2.)

A Semicircle is half an entire circle as ABG.

A Segment of a circle is a figure contained between a right line called a chord as EF, and any part of the circumference EDF, or EBF, called an arch.

SEC. I. DEFINITIONS.

5

A Sector of a circle is made by two radius's as CG and CF, and part of the circumference FG.

A Quadrant of a circle is the fourth part, or that contained between the lines CA and CB, and part of the circumference AB.

Every circle is supposed to be divided into 360 equal parts called degrees ; a degree into 60 equal parts called minutes ; and a minute into 60 equal parts called seconds, &c.

V. An Angle is the inclination or meeting of two right lines as CG and CF, or CA and CF, in the point C. (Fig.2)

A right Angle is made by one right line BC, standing on another AG, and making the angles on both sides equal, as the angle ACB is equal to the angle BCG, each being a right one, and containing the

fourth part of a circle or 90 degrees ; and the line BC is said to be perpendicular to AG.

N. B. Where an angle is expressed by three letters (as above) the middle one always denotes the angular point.

An acute Angle is less than a right angle, as the angle FCG.

An obtuse Angle is greater than a right angle, as the angle ACF. (Fig. 2.)

VI. A plain Triangle is a figure or superficies bounded by three right lines ; and of which there are the following sorts.

First, an equalateral triangle is that whose bounds or sides are all equal as (Fig. 3.)

Second, an isosceles triangle is when only two sides are equal as (Fig. 4.)

Third, a scalene triangle is when all the three sides are unequal as (Fig. 5.)

SEC. I. DEFINITIONS.

7

Fourth, a right angled triangle is that which has one right angle as ACB (Fig. 6.) whereof the side AB opposite to the right angle is called the hypotenuse.

VII. A Square, is a four-sided figure whose sides are all equal; and its angles right ones, or each equal 90 degrees. (Fig. 7.)

VIII. A Parallelogram, is a four-sided figure, whose opposite sides only are equal and angles right ones. It is also called a Rectangle. (Fig. 8.)

IX. A Rhombus, is a four-sided figure having all its sides equal, and the opposite angles equal but not right ones, two being acute and two obtuse. (Fig. 9.)

B 4

X. A

X. A Rhomboides, is a four-sided figure having only it's opposite sides equal, and it's opposite angles equal but not right ones. (Fig. 10.)

XI. A Trapezium, is a four-sided figure, having all it's sides as well as angles unequal. (Fig. 11.)

Note, If a right line be drawn from the opposite angles of any four-sided figure, it reduces it to two triangles, and such line is called a diagonal, as the dotted line AC. (Fig. 11.)

XII. A Polygon, is a figure having more than four sides; and if the sides and angles are unequal it is called an irregular polygon; but if the sides are all equal it is called a regular polygon; and are named according to the number of it's sides as follows.

A Pen-

SEC. I. DEFINITIONS.

9

A Pentagon, has five equal sides and angles each angle at the circumference is 108 degrees, and at the center 72 degrees.

A Hexagon, has six equal sides and angles, each angle at the circumference is 120 degrees, and at the center 60 degrees.

A Heptagon, has seven equal sides and angles, each angle at the circumference is 128,57, and at the center 51,43 degrees.

An Octagon, has eight equal sides and angles, each angle at the circumference is 135, and at the center 45 degrees.

A Nonagon, has nine sides and angles, each angle at the circumference is 140, and at the center 40 degrees.

A Decagon, has ten equal sides and angles, each angle at the circumference is 144, and at the center 36 degrees.

An Undecagon, has eleven sides and angles, each angle at the circumference is

147,27, and at the center 32,72 degrees.

A Dodecagon, is a figure of twelve equal sides and angles, each angle at the circumference is 150, and at the center 30 degrees.

XIII. An Ellipsis (or oval), is a plain figure bounded by one curved line called the circumference or periphery ; and is produced by the section of a Cone. And may be produced also by the motion of two right lines about two fixed points, (called focufes) these right lines are continually varying their lengths, and as one lengthens the other shortens and vice versa ; making their sum every where the same, and equal its longest diameter.

The longest diameter is called the Transverse diameter or axis, as AB. Fig. 12.)

The shortest diameter DE, is called the conjugate

conjugate diameter or axis ; and these two diameters intersect each other at right angles in the middle of each or center of the ellipsis.

The focuses are two points in the transverse diameter equally distant from the center as Ff , and on which one end of the moving lines are fixed while they describe the periphery.

SECTION II.

Containing some useful Geometrical Problems.

PROBLEM I.

To erect a perpendicular at a point near the middle of a given right line.

LET AB , be the given line, and C the point on which it is required to erect the perpendicular. (Fig. 13.)

With

With any convenient opening of the compasses and one foot in C turn the other to the points D and E; then setting one foot of the compasses in D, and with any opening of them at pleasure describe an arch; also with the same extent of your compasses and one foot in E, intersect that arch in the point F. Draw the line CF, and it will be the perpendicular required.

P R O B L E M II.

To erect a perpendicular at the end of a given line.

Let the given line be AB. (Fig. 14.) Set one foot of the compasses at the end of the line or point B, and open the other to any convenient distance, and describe the arch CDE; with the same extent and one foot in C make the intersection at D, also with the same extent and one foot in D describe

scribe the arch EF, fill with the same extent and one foot in E describe the arch DF. Then through the intersection at F, draw the line BF and it will be the perpendicular required.

A SECOND METHOD.

With any convenient opening of the compasses and one foot in B (Fig. 15.) turn the other to any point C above the given line AB, and on this point describe the arch DBE, greater than a semicircle, lay a ruler from D to C, and it will cut the said arch in E, draw BE and it is done.

PROBLEM. III.

From a given point to let fall a perpendicular on a given right line.

Let C be the point above the given line AB, (Fig. 16.) Then with one foot of the
com-

compasses in C describe an arch to cut the line AB in two points D and E, with one foot of the compasses in D and E severally make the intersection at G; lay a ruler from G to C, and draw FC and it will be the perpendicular required.

P R O B L E M IV.

To bisect or divide a given right line into two equal parts.

Suppose AB (Fig. 17.) the given line. Then with any opening of the compasses greater than half the given line and one foot in A describe an arch CD, with the same extent and one foot in B describe another arch CD intersecting the former in the points C and D; lay a ruler to the points C and D and draw the right line CED, and it will divide the given line into two equal parts at the point E.

P R O

PROBLEM V.

To bisect or divide any given angle into two equal parts.

Let ABC (Fig. 18.) be the given angle. Set one foot of the compasses in the angular point B, and with any opening of them describe the arch DE; then with one foot in D and E severally describe two arches intersecting each other at F: draw BF and it is done.

PROBLEM VI.

To make or lay down an angle equal to a given right lined angle.

Let the given angle be ABC, (Fig. 18.) Draw any right line DE (Fig. 19.) then with the extent BD (Fig. 18.) and one foot of the compasses in E describe the arch AB (Fig. 19.) take the length of the arch DE, and

and set it from A to B, and through the point B draw the line EF; so shall the angle DEF be equal the given angle ABC.

P R O B L E M VII.

To divide a given right line into any proposed number of equal parts.

Let AB (Fig. 20.) be divided into five equal parts. From the end A draw the line AC out at pleasure, making any angle with AB; from the other end B, draw BD, making the angle ABD, equal the angle BAC, (by Prob. 6.) From the points A and B, set off on the lines AC and BD any five equal parts as Aa, ab, bc, cd, de; and Bf, fg, gh, hi, ik; draw the lines Ak, ai, bh, cg, df, and eB, and they will divide the given line as required in the points 1, 2, 3, 4, 5.

O T H E R.

OTHERWISE.

Let EF (Fig. 21.) be a line given to be divided into six equal parts.

Draw an occult line AB of any convenient length, and upon it from A set off as many equal parts as the given line EF ought to have; fix for example as from A to C, which mark with 1, 2, 3, 4, 5, 6.

Take the extent AC in your compasses and with one foot in A and C severally make the intersection D. From D to the several points A, 1, 2, 3, 4, 5, 6, draw the lines DA, D₁, D₂, D₃, &c.

Take the given line EF in your compasses and set it from D to *e* and *f*, and draw the line *ef*, which will be equal to the line EF, and divided into six equal parts as required.

SCHOLIUM.

Hence we have a method of dividing several right lines of different lengths into the same number of equal parts at once.

P R O B L E M VIII.

To draw a line parrallel to a given right line, and to pass through any assigned point.

Let the given line be AB (Fig. 22.) and the point assigned be C. Take in your compasses the least distance from C to the given line AB, (continued if needful) with this extent and one foot of the compasses on or near the end of the line AB that is farthest from C, describe the arch D; a right line drawn from the point C to the top of the arch D, will be parallel to AB as required.

P R O B.

PROBLEM IX.

Having three right lines given, so that the sum of the two shortest shall be greater than the third, to construct a triangle.

Let AB, BC, and CA (Fig. 3.) be the three given lines. Lay down the line AB, take the line BC in your compasses and setting one foot in B describe the arch C, also with the line CA in the compasses and one foot in A make the intersection at the point C, join CA, and CB, and they will form the triangle required.

PROBLEM, X.

Having any given right line to construct a square.

Let the given line be AB, (Fig. 7.) Upon the point B erect the perpendicular BC (by Prob. 2.) making it equal to AB; with

C 2

the

the extent of AB in your compasses and one foot in A and C severally, make the intersection at D, and draw the lines DA, and DC, and it's done.

P R O B L E M XI.

Having any two right lines given to construct a parallelogram.

Let the given lines be AB and BC (Fig. 8.) Upon the point B at the end of the longer line AB, erect a perpendicular (by Prob. 2.) making it equal to the shorter line BC; then with the extent of the line AB in your compasses and one foot in C describe the arch D, take the shorter line BC in your compasses, and with one foot in A cross the former arch in the point D, and draw DA, and DC, and it is done.

P R O B.

PROBLEM XII.

Upon any given right line to construct a rhombus.

Let AB (Fig. 9.) be the given line. With the extent of AB in your compasses and one foot in B describe the arch DC, with the same extent and one foot of the compasses in A make the intersection D; still with the same extent and one foot of the compasses in D, make the intersection C; draw the right lines AD, DC, and CB, and it is done.

PROBLEM XIII.

Having any two right lines given to construct a rhomboides.

Let the given lines be AB, and AD, (Fig. 10.) Take in your compasses the shorter line AD, and set it on the longer

C 3

line

line AB from A to X, with the same extent and one foot of the compasses in A and X severally make the intersection D; with the same extent and one foot of your compasses in B, describe an arch C; with the extent of AB in your compasses and one foot in D intersect the former arch in C; draw the lines AD, DC, and CB, and it is done.

P R O B L E M XIV.

Having the diagonal and four sides given to construct a trapezium,

Let AC be the given diagonal, and AB, BC, CD, and DA the given sides, (Fig. II.) Having drawn an occult line AC, and made it equal to the given diagonal; take the extent AB in the compasses and with one foot in A describe an arch at B, with the extent of BC in the compasses and one foot in

in

in C intersect the former arch in B; draw the lines AB and CB; take the extent AD in the compasses and with one foot in A describe an arch at D, with the extent CD in the compasses and one foot in C intersect the arch at D; draw AD and CD, and it is done.

O T H E R W I S E.

By having the four sides and one angle given to construct a trapezium.

Make the line AB (Fig. 11.) equal its given side, and on the point A make the angle BAD equal the given angle (by Prob. 6. or 21.) and make the line AD equal its given side; then with the extent of the line BC in your compasses and one foot in B, describe the arch C, also with the line CD in your compasses and one foot in D,

C 4

intersect

intersect the arch in C, and join CB and CD with right lines, and it is done.

P R O B L E M XV.

In any given circle to describe a regular polygon.

Let ABCDEFGH, (Fig. 23.) be the given circle, in which it is required to make a regular heptagon.

Draw the diameter AD, and divide it (by Prob. 7.) into as many equal parts as the proposed polygon hath sides, (which in the present case is seven); take the extent AD the circle's diameter in your compasses, and with one foot in A and D severally make the intersection at X; from X draw a right line always through the second division in the diameter, and it will cut the periphery of the opposite semicircle in the point B; draw the right line AB and it
will

will be the side of the polygon required ; which being set round within the circle from B to C, and from C to E, &c. compleats the polygon.

P R O B L E M XVI.

To construct any regular polygon each of whose sides shall be equal a given right line.

With any extent at pleasure in your compasses as a radius and one foot in C (Fig. 24.) describe a circle, in which find (by Prob. 15.) the side An of a polygon of AB the same number of sides with that required, (suppose six); draw CD parallel to AB, on which set the extent of half the given line from C to D, at D erect the perpendicular DE, cutting CA (extended if necessary) in the point E; with the extent of CE in your compasses and one foot in C describe a circle, in the circumference of

of which apply continually the given side, and it will form the polygon EFGHIK as required.

O T H E R W I S E .

Having described the circle, (Fig. 24.) make the angle ACB equal the angle at the center of the required polygon, (see Defin. 12.) which is always equal to 360 degrees divided by the number of the polygon's sides ; and draw the right line AB, and parallel thereto draw CD making it equal to half the given side, erect the perpendicular DE cutting CA in E, so shall CE be the radius of the required polygon's circumscribing circle ; in the circumference of which repeat the polygon's side.

C O R O L L A R Y.

It may be proper to observe that any polygon may be inscribed in a circle by

this construction with more accuracy than by the 15th problem; that being that of RENALDINES, which may do very well for constructing small figures on paper; but will not answer in all cases in large works, or where great exactness is required.

PROBLEM XVII.

To find the center of a circle that shall pass through any three given points, not lying in a right line.

Let A, B, D, be the points given (Fig. 25.) join AB and BD with right lines, then bisect those lines (by Prob. 4.) and the point where the bisecting lines meet as at C, will be the center of the circle required.

COROLLARY I.

Hence any triangle may be inscribed in a circle by bisecting two of its sides.

COROL.

COROLLARY II.

Having only the arch of a circle, the circle may be perfected.

PROBLEM XVIII.

To construct an ellipsis by having the transverse and conjugate diameters given.

Let AB and DE (Fig. 12.) be the given diameters.

Having drawn the transverse AB, and its conjugate DE, bisecting each other perpendicularly in the center C; with the extent of AC in your compasses and one foot in D, intersect the transverse diameter AB, in the points F and f , which will be the foci of the ellipse.

Take any point (1) in the transverse between C and F, and with the extent of A1 in the compasses and one foot in F and f severally, describe the arches Gg, also with
the

the extent B_1 and one foot in f and F , intersect those arches in $\dot{G}$ and g ; and so will you have four points in the ellipsis's periphery.

Take any other point (2) in the transverse between C and F , and with the extent of A_2 in the compasses and one foot in F and f severally describe the arches hh ; also with the extent B_2 and one foot in f and F , intersect those arches in the points h and h ; and so will you have four more points in the ellipsis's periphery.

And thus by assuming several points 1, 2, 3, &c. in the transverse, there will be found as many points in the curve as you shall chuse; through all which with an even hand draw the ellipsis's periphery.

OTHERWISE.

Having found the focuses F and f , take a thread that shall be just the length of the

transverse diameter, when two loops are made on it, viz. one at each end; then stick a pin in each focus, making them fast by driving them a little into the table; put a loop over each pin and with a pencil or pen stretch the thread, and moving the pencil round from A to D and B, it will describe half the periphery ADB; then slip the thread between the pins towards E, and stretching the thread with the pencil begin at B and move the pencil (keeping the thread stretched) round by E to A and it will compleat the ellipsis's periphery.

And in the same manner, with two stakes and a chord, may an elliptic fountain, grasse plat, &c. be marked out in gardens, pleasure grounds, &c.

P R O B-

P R O B L E M XIX.

To make a trapezium or a triangle equal to any given pentagon.

Let ABCDE, (Fig. 26.) be the given pentagon. Continue one of the lines each way at pleasure as AB; then draw DA, and parallel thereto draw EF, cutting AB (continued) in F, join DF with a right line; then the trapezium DFBC, will be equal to the pentagon ABCDE.

Draw DB, and parallel thereto draw CG cutting AB continued in G; join DG with a right line, and the triangle DFG, will also be equal to the given pentagon ABCDE, as required.

P R O B L E M XX.

To make a trapezium equal to any given irregular figure of six, seven, &c. sides.

Let

Let ABCDE (Fig. 27.) be a seven-sided figure given.

Having continued one of its sides, as AB, draw BD, and parallel thereto CH; join DH and it reduces the two lines BC and CD to one straight line DH. Draw AF, and parallel thereto GI; join FI, and it reduces the two lines AG and GF to one FI. Draw EF, and parallel thereto FK; join EK and it reduces the three lines AG, GF and FE to one straight line EK; and the trapezium EDHK is equal to the seven-sided figure ABCDEFG, as required.

P R O B L E M XXI.

To make or lay down an angle of any proposed number of degrees by the scale of chords.

Take in your compasses the extent of the first 60 degrees from a line of chords, and with one foot in A (Fig. 28.) describe
the

the arch BDE ; extend the compasses from the beginning of the line of chords to the proposed number of degrees if less than 90 ; and set that extent upon the arch from B to D, so shall BAD be the angle required.

But if the angle be more than 90 degrees take the extent of half the number of degrees and turn the compasses twice over upon the arch as from B to E, and BAE shall be the angle required.

P R O B L E M XXII.

To divide any given arch of a circle into any proposed number of equal parts suppose the fourth part of a degree or 15 minutes.

If the given arch be less than 60 degrees, make it equal to 60 by applying the radius of the given arch upon the said arch continued ; as the radius AB (Fig.

D

28.)

28.) being taken in the compasses and applied from B to D, so will BD be an arch of 60 degrees. This arch being bisected gives the arch of 30 degrees, and 30 bisected gives the arch of 15 degrees, and this being bisected gives $7\frac{1}{2}$ degrees, and this bisected gives $3\frac{1}{4}$ degrees; take this extent of $3\frac{1}{4}$ degrees in the compasses and with one foot in B describe a small arch, also with the extent of AB and one foot in A intersect the small arch in C, and draw BC continuing it a little beyond C. Now BC being the chord of $3\frac{1}{4}$ degrees only, it will not differ sensibly from the arch itself; for the (natural) sine and tangent of an arch of 4 degrees are very near of the same length; the tangent being ,06993 and the sine ,06976 (to the radius 1.) and their difference is ,00017 not the two ten thousandth part of the radius.

Draw

Draw any line BF and parallel thereto (by Prob. 8.) through C draw CG; then with any small opening of your compasses set one such extent on the line GC, from C to the right hand as to *a*, and three on the left to *b*; take the extent of *a b* (equal to the four small divisions) in the compasses and set off three equal divisions on CG from *b* towards G as the points *c, d, e*, and from B towards F set four as the points *f, g, h, i*; join *ia, hb, gc, fd*, and *Be*, and they will cross BC in four points, each equal to one degree, the first crossing it in the point X, so will BX be equal to 4 degrees, and DX equal to 64 degrees: Now this arch DX or 64 degrees may be bisected continually till you get the fourth part of a degree or 15 minutes, which is agreeable to the conditions of the problem.

If the half of DX or 32 degrees be set

D 2

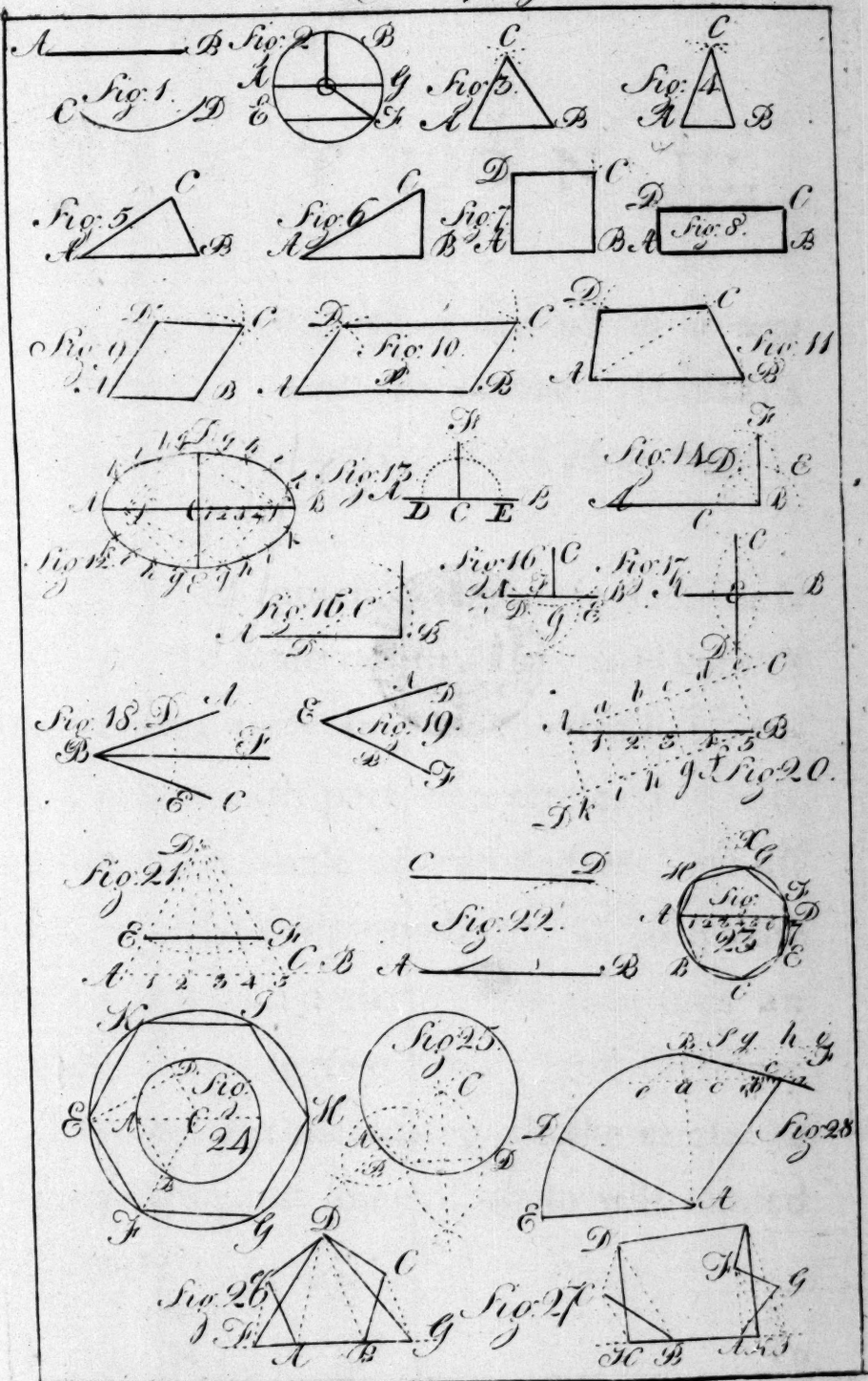
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from D to E, then may the arch DE be bisected also; and thus you will have an arch of 96 degrees divided into every 15 minutes, provided the radius of the arch be large enough to admit of such division.

But the bisection for finding the arch of 4 degrees may be saved, by multiplying .06993 the tangent of 4 degrees by the radius of the arch in inches, and the product taken from an inch diagonal scale and set from B to X.

Perhaps this method will be found preferable to one published by Mr. BIRD, and for which he received a large premium from the board of longitude

Plate 1 to face Page 36.





S E C T I O N III.

*Containing the description and use of a new
Catoptric Sextant, or improv'd HADLEY'S
Quadrant in the art of Land Measuring.*

THE Instrument I shall here describe is of the same construction as HADLEY'S Quadrant, only in this the limb is enlarged to a sixth part of a circle, and will measure an angle of 120 degrees; whilst that of HADLEY'S quadrant, is only the eighth part of a circle, and measures an angle of 90 degrees only; and therefore insufficient for measuring all the angles of a trapezium, as some of them will exceed 90 degrees.

In (Fig. 1. Plate 2.) ABC represents the frame of this instrument, H, and I, two braces to keep it from casting or warping: the limb AB is an arch of 60 degrees, whose chord is equal the length of the radii CA, or CB, and is divided into 120 equal parts, and in measuring angles with this instrument answers to 120 degrees; and are numbered, 0, 10, 20, 30, &c. to 120, again, each degree is sub-divided into minutes. The index CD is moveable round the center of the instrument and that part of it which slides over the graduated arch AB is open in the middle, with VERNIER'S scale on the lower part of it: and underneath is a spring that keeps the index to any part of the limb it is set to.

E, is a speculum or piece of looking-glass, set in a frame of brass or wood, and fixed on the index, perpendicular to
the

the plane of the instrument, and to coincide exactly with the center thereof and a line supposed to be drawn down the middle of the index.

F, is another small glass set in a frame nearly the same as the other, and fixed on the side of the instrument as in the figure, also perpendicular to the plane thereof: the lower half or that next the instrument only is quicksilvered; there being an opening in the frame against the upper half to view the horizon or any other object through, and is called the horizon glass.

G, is a sight vane, fixed in the opposite side to F, and should be set so near the center of the instrument, that the visual line from it to the horizon glass at F may but just pass by the great speculum E: in this vane is a small hole to guide the eye

to the horizon glass: this hole must be at the same distance from the plane of the instrument, as is the edge of the quicksilvered part of the horizon glass.

And in order to make this instrument useful in taking of altitudes at land without a visible horizon; I have added the spirit level K, which is fixed to a brass plate by two small screws; one is a central one, on which the level turns a little way backwards or forwards, for adjusting it, the other goes through a small spring, and serves to secure the level when it is rightly adjusted: the brass plate with the level, being fixed with screws to the back of the instrument so as the level may fall in between the two sides thereof; and so is very convenient to be view'd; and yet secure from danger in carriage.

By means of this level being properly adjusted, we may discover our true horizon, and compare the altitudes of objects therewith; and for this purpose there must be a black line drawn across the glass E, exactly over the center, and perpendicular to the plane of the instrument: this line will appear very distinctly in the horizon glass, and seem as if it was drawn thereon; or which I think is better, a line may be drawn along the middle of the horizon glass perpendicular to the plane of the instrument; and then the sextant being held or fixed in a vertical position so as the bubble of the level may rest in the middle of its tube; a line passing through the center of the hole in the sight vane, to the black line on the horizon glass, will be parallel to the true horizon.

Before

Before I proceed to the use of this instrument, it may be necessary to explain the Vernier or scale for shewing the minutes of a degree; which by some is also called a Nonius: and is beyond dispute, the most excellent and elegant of all others: it is also very easy to be understood, and used, when rightly considered: for since each degree is divided as aforesaid, into three equal parts, there will be twenty-one of those parts in seven degrees, each of which contains twenty minutes: then if on that part of the index called the Vernier, we take a length just equal to seven degrees, and divide it into twenty equal parts, it is evident that since twenty of these answers to twenty-one on the limb; each one of these, will exceed each one on the limb by $\frac{1}{20}$ part, that is by one minute: therefore two on
the

the vernier, will exceed two on the limb by two minutes, three on the vernier by three minutes, and so on. Consequently if the lines at the beginning of the divisions on the vernier be any where between two divisions on the limb, there will be a coincidence of one division on the vernier, with one on the limb somewhere; and by observing where that coincidence is, you will see at the same time how many minutes the index has passed the last division on the limb: thus if the vernier coincides at the third division from the beginning or that marked $\frac{0}{20}$ it shews the index has passed the last division by three minutes; if the coincidence be at the sixth line on the vernier, then is the index six minutes beyond the last on the limb, and so of the rest.

If

If two lines on the limb should fall within two lines of the vernier (as will sometimes happen) then that shews you must reckon half a minute or thirty seconds more than the left hand division denotes.

If the first line or index of the vernier has moved over a space less than half a division on the limb; then the coincidence will be on the right hand in the vernier: if it has moved over more than half, it will be found on the left hand; if over just half, the coincidence will be at ten on the vernier.

If the beginning of the index fall between twenty and forty minutes on the limb the vernier shews the minutes to be added to twenty: if it falls beyond forty minutes the vernier shews how many minutes must be added to forty.

But

But to make this still plainer, I have added Figure 2. in which AB represents a part of the graduated limb of the instrument; and D that part of the index having the vernier; and suppose in making an observation, the beginning of the vernier be found between twenty-six degrees forty minutes, and twenty-seven degrees, as in the figure; then looking for the coincidence of the divisions on the limb and vernier, I find it to be at the sixth division of the vernier; therefore the angle is shewn to be twenty-six degrees, and forty-six minutes.

It may be proper to observe that figure 2. was taken from an instrument of two feet radius; but some may choose one of a smaller size, as of one foot radius, which is more portable and yet sufficient in most cases for the use of the land surveyor.

But

But as this is a radius too small to admit of a scale to shew every minute, I shall shew how to construct one for shewing every five minutes. In this case the limb of the instrument need only to be divided into 120 primary divisions or degrees: then taking on the vernier a space equal to thirteen of these degrees on the limb; and dividing it into twelve equal parts, each of these will be five minutes, and twelve of them equal to sixty minutes or one degree: and if on the vernier you mark the middle division with $\circ$, and the third division (from the middle) to the right and left hand respectively with 15 and 45; and the sixth or last division on each side from the middle one with 30, it is plain that such a scale will shew an angle to every five minutes.

Having

Plate 2 to face page 46.

Fig. 1.

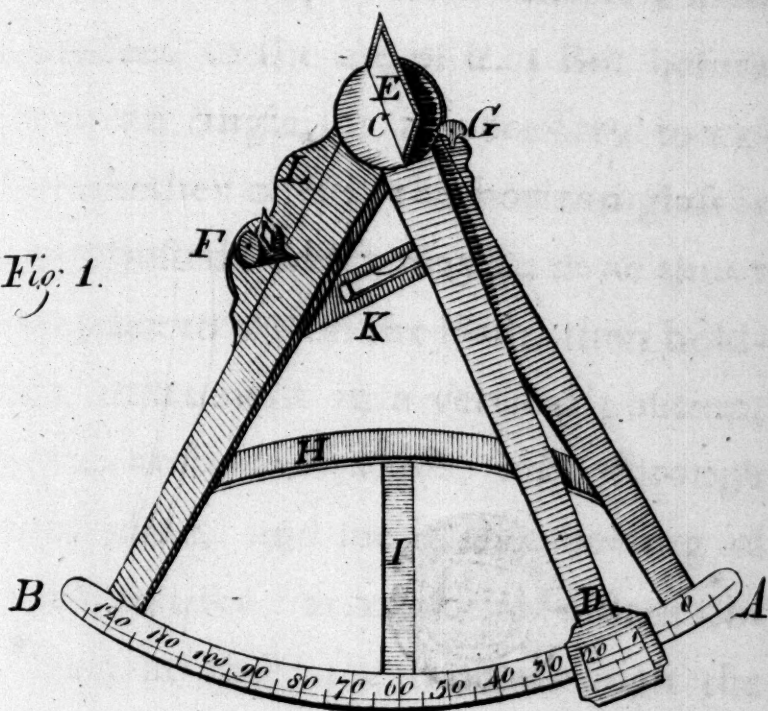
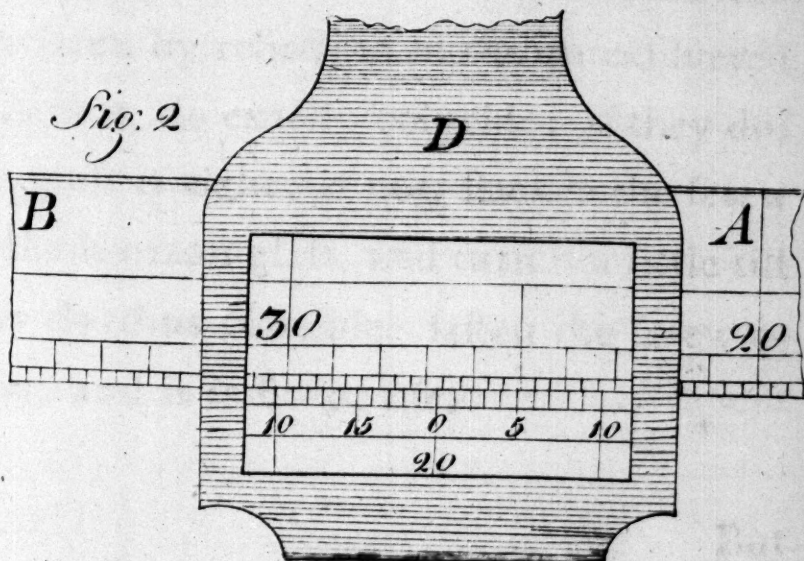
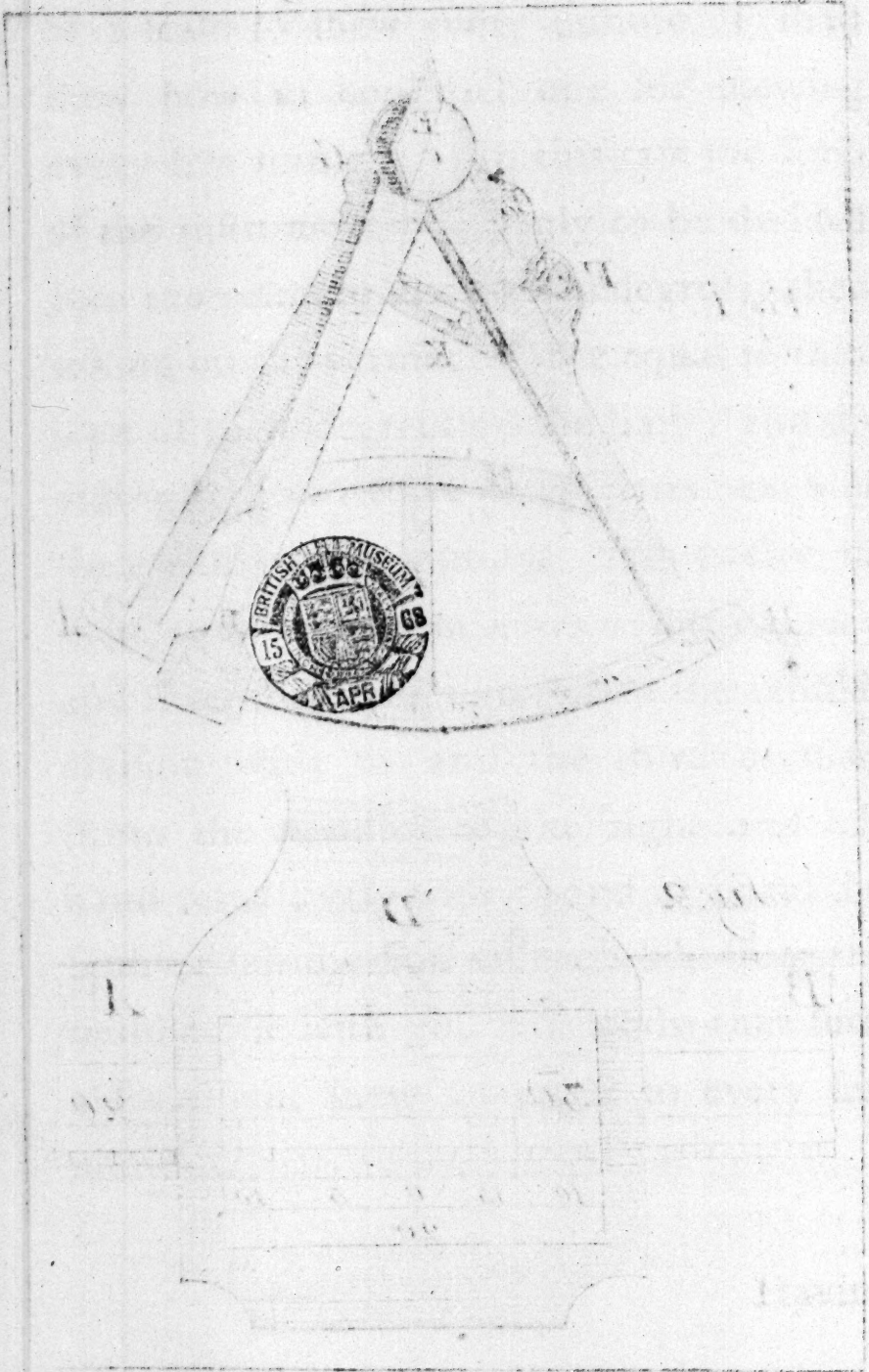


Fig. 2



White & Jackson 18



Having given (I think) a sufficient explanation of this capital instrument; I shall now proceed to the use of it. But before we take an angle, it is necessary to examine whether or no, the horizon glass is rightly adjusted; which may be done thus: set the index to 0° . on the limb, then holding the instrument in a vertical position, with the arch downwards, look through the sight vane, and see if the meeting of the land and sky, or any other distant object, seen at the same time through the clear part of the horizon glass and the same seen by reflection in the quicksilvered part of it, do exactly coincide; if they do, the glass is right: if not, slacken the screw of the horizon glass, and turn it a little till they do thus coincide, fasten the screw again, and it is fit for use.

But

But this trouble of adjusting the glass may be saved, by looking through the sights, and moving the index till the objects do thus coincide; then see how many minutes or degrees and minutes, the index falls short of, or is advanced before, the first division on the limb, or that mark'd 0° . if the index falls short, add the minutes; but if advanced, subtract them, to or from the angle found by observation, and the result will be the true angle.

To take a vertical angle, or angle of altitude, as suppose of the tower BC (Fig 3. Plate 3.)

First standing at A hold or fix the sextant with the arch downwards, so that the air bubble may rest in the middle of its tube; look through the sight vane at the tower, and where the black line of the horizon
then

rizon glafs cuts it make a mark as at E; then will the vifual line DE, from the eye at D, to the mark at E, become both a true and vifible horizon. Then holding up the fextant and looking through the fight vane, at the top of the tower C, make the direct object and reflected image coincide, then fee how much the index falls before the divifion on the limb marked 0° . fuppofe five minutes, now looking at the top C again, flide the index forwards till the image of the top at C is depressed or brought down to a level with the vifible horizon DE, and it will appear at F at the fame diftance from the eye, as is the object C: obferve the degrees and minutes cut by the index, fuppofe $41^{\circ}, 25'$; out of which deduct $5'$ for the coincidence before found, and there remains $41^{\circ}, 20'$ for the angle of altitude.

E

And

And in the same manner may the altitude of any other object be found, as the sun for example, the instrument being adjusted, hold it in a vertical position and slide the index forward till the image of the sun appears at G, and seems to be bisected by the mark at E, then will the index shew the altitude of the sun's center. If you have not a conveniency for making the visible horizon E; bring the sun's image as near the horizon as you can guess, then holding the instrument so that the bubble may rest in the middle of its tube, move the index till the solar image is bisected by the black line on the horizon glass, then the index shews the altitude as before.

Note, If the reflection of the solar rays be too bright for the eye, a dark or smoaked glass may be placed in the hole at L, (Fig. 1.)

To

SEC. III. OF THE SEXTANT. 51

To measure an horizontal angle with the sextant; suppose the angle BAC (Fig. 4.) or that which the boundaries or two sides of a field make with each other.

First set up marks or whites at B and C, so as to be seen when standing at the corner, or angular point A. Set 0° on the index to $0'$ on the limb; then holding the instrument in an horizontal position, and looking through the sight vane and horizon glass, at the white at B, see if the same seen by reflection appears exactly in a perpendicular line just under it; if not, move the index a little backwards or forwards, till they do thus coincide; and observe how many minutes it falls short, or is advanced; suppose it's advanced $10'$: then holding the instrument as before, look through the sights at the white at B, observe the image of the same

in the quicksilvered part of the horizon glass; then moving the index gently forward with your left hand, and the reflected image will advance from B towards C; on which keeping your eye, and continuing the motion of the index till the reflected image coincides with or seems to touch the white at C; which may be easily done, by twisting the instrument a little up and down: and then will the said image appear at D, at the same distance from A, as is the object B: see what degrees and minutes are cut by the index; as suppose $75^{\circ}, 20'$, from which subtract $10'$ for the coincidence or correction of the instrument, and the remainder $75^{\circ}, 10'$, is the measure of the angle BAC, required.

And thus may any angle less than 120 degrees be measured with the utmost ease, exactness and expedition. But it may so
happen

happen (though very rarely in practice) that the measure of an angle greater than 120 degrees may be wanted; and in order to measure such an angle, let A B and A C, (Fig. 5.) be the two sides of a field, whose included angle at A is sought. Having set up whites at B and C, (as before directed) and found the angle BAC to be greater than the instrument will measure at once, set up another white beteen B and C, at a good distance from A, as at G; or fix upon some remarkable object at a distance, between B and C, as suppose the tree D; then setting the index to 0° . observe the object and image at B to coincide when the instrument is held nearly horizontal; bring the image at B, (by sliding the index forwards) to coincide with the white G, or the tree at D; and see what degrees and minutes are cut by the index,

suppose $84^{\circ}, 15'$, and such is the angle BAD, or BAG. Set the index to 0° again, and bring the image of the white G, or tree D, to coincide with the white C: then will the index shew the angle GAC, or DAC, suppose $73^{\circ}, 45'$, to which add the angle BAD, and the sum will be equal BAC, $158^{\circ}, 00'$ the angle sought.

Note, if the object and image at B had not coincided, when the index was at 0° the angles BAD, and DAC, must have been corrected as in the foregoing examples. Let us suppose at the coincidence of the image and object B, the index fell $3'$ short of the 0° , then must $3'$ be added to $84^{\circ}, 15'$, and the sum $84^{\circ}, 18'$ is the correct angle BAD. The same regard must be had to the angle DAC, and in such case the sum of the corrected angles will be the true angle BAC as required.

This

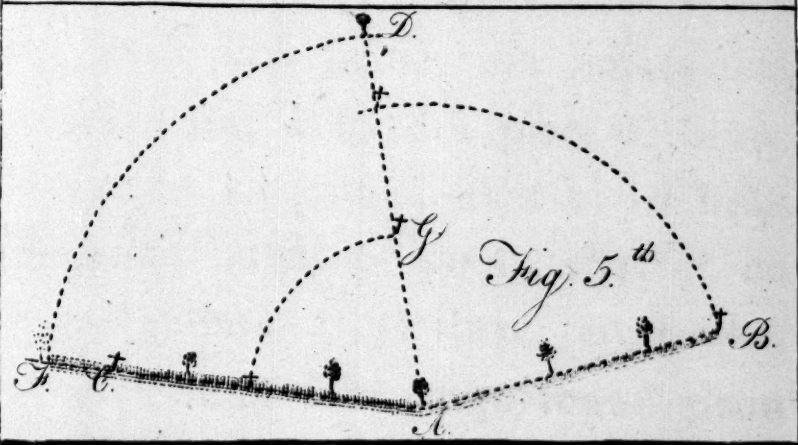
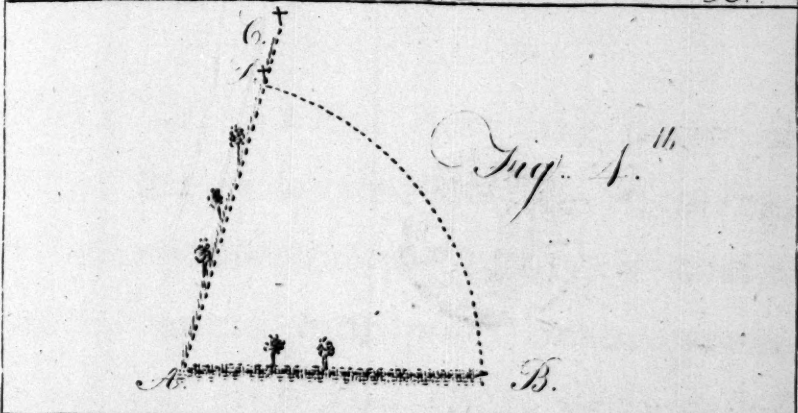
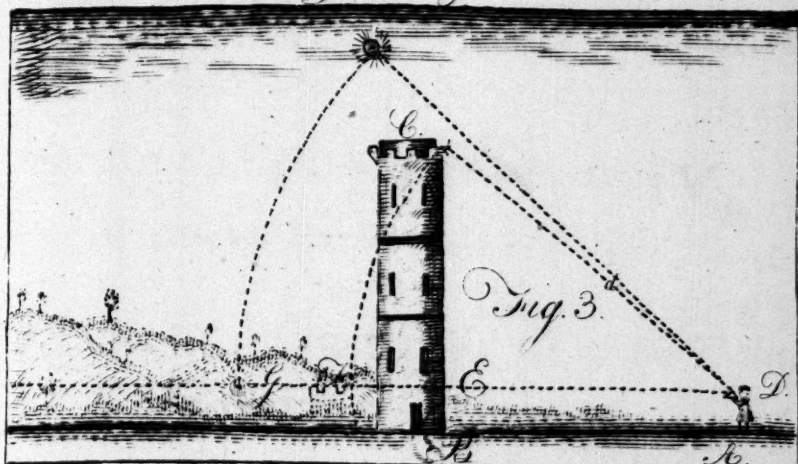
✍ This instrument I would recommend to every practical surveyor, having from sufficient experience, not only found it to be ten times more portable and expeditious, but equally exact with the best modern theodolite; though unassisted with telescopic sights: not but such sights may be easily applied to this curious instrument. Another recommendation is it's cheapness, as a very good one may be made for a couple of guineas, whilst some of our modern theodolites are valued at twenty. It may be hung by a belt over the shoulder, and be used without taking of with the utmost ease and expedition, being no ways cumbersome; and consequently two men will do nearly as much work in a day with this instrument, as three can do with a theodolite; it being

one man's business to carry the theodolite.

The particular uses of the sextant will be exemplified in the following parts of this work.

SECTION

Plate 3 to face Page 56.





SECTION IV.

*Containing the description and use of the chain,
off-set-staff, and arrows.*

THE chain I shall here take notice of is that invented by Mr. GUNTER, and commonly called GUNTER's chain. It is in length four poles or 22 yards, and divided into 100 equal parts called links; each link being 7,92 inches: at each tenth link is fixed a piece of brass, with notches or points; that at 10 links having one notch or point, that at 20 links two points, at 30 three, at 40 four, at 50 or middle is a large round plain one, at 60 is the same as 40, and 70 the same

same as 30, &c. By this means it is easy to count the links from either end of the chain. The first link at each end is usually made into a large ring or bow, for the ease of holding it in the hand.

This is the best instrument yet invented for measuring lines in the field.

The off-set-staff is best made of deal or some such light wood, and shod at one end with iron, and made sharp or pointed a little to stick into the ground when there is occasion : at the other end may be a small notch, it being of use for putting the end of the chain through hedges, &c. this staff must be just 10 links in length, or 6,6 feet ; and divided into 10 equal parts or links : the first link may be divided into 10 equal parts, or decimals of a link,

a link, which may be useful in some nice cases.

The arrows are ten small sticks, best made of ash, and about 18 inches long, shod and pointed at one end so as they will easily stick into the ground: or instead of these, you may have 10 iron wires about a foot long, exclusive of three or four inches, that are bent down, in order to hang them to a belt: at the ends bent down may be fixed bits of cloth to distinguish them from the grass or corn, &c.

DIREC-

DIRECTIONS for using the CHAIN.

P R O B L E M I.

To measure a right line all visible and accessible from each end by the chain.

Having set up marks at each of the places betwixt which you are to measure; call the place you begin at your first station, and the place you measure to, your second station; then having stretched out the chain, examine it with the off-set-staff, and see it be of a right length, and the links all straight and right: then let your assistant take the ten arrows in his left hand, (or which is better, in a pouch) and the end of the chain in his right, and proceed towards the second station, whilst you hold the other end of the chain close

to

to the first station; direct him to stick down an arrow at the end of the chain, exactly in the station line, so that your first station, the arrow, and second station, may be all in one right line; proceed another chain's length, and being come to the first arrow, hold your end close to the arrow, and direct your assistant to stretch the chain, and at the end thereof to stick down a second arrow in the station line as before; and if he looks back at the first station, he will see the two arrows and the station all in one right line, if the arrows were rightly placed; and thus you may correct each other. This being observed, you take up the first arrow and proceed to the second, giving your assistant directions to put down a third arrow, and then you take up the second. And thus you proceed to the end of the line, unless

unless it exceeds 11 chains ; if so, being come to the tenth arrow, direct the leader to stretch the chain exactly in the station line ; then return him the ten arrows, and write in your field book 1 change ; the leader sticks down an arrow at the end of the chain, and then you proceed as before till you have measured to your second station ; then see what number of links are contained between the last arrow and the station ; suppose 73, and the number of arrows you have will be the number of chains if less than 11, otherwise you add 10 for every change, as suppose one change and 8 arrows, this makes 18 chains. Then enter the chains and links in your field book with a comma between them thus 18,73. But if the links are under 10, a cypher must be added between the chains and links, suppose the above had been only

only

only 7 links more than the chains, it must have been set down thus 18,07; the links being always decimals of a chain. Remember always as soon as you have measured a line, and set down the dimensions, to examine the arrows and see you have them all; for if one should be missing the last line will be doubtful, and must be re-measured before you proceed.

Note, It is usual to allow five links from the stem of the quickset hedge, for the breadth of the ditch, except the custom or agreement is otherwise; but the custom of the place generally is the surveyor's rule; but against roads or commons, it is usual to allow six links.

P R O B L E M II.

To measure a right line, part of which goes through a hollow, the rest visible from both ends, and all of it accessible, the hollow not very deep nor the sides steep, by the chain.

Begin and measure as directed in the last problem, till you are likely to lose sight of your second station, then let your assistant take the off-set-staff and go to the other side of the hollow, and direct him to place it exactly in the station line, and so as to be seen from the bottom of the hollow: measure to the staff; and from thence to the end of your line.

P R O-

P R O B L E M III.

To measure a right line, all accessible, but part of it not visible from both ends by reason of a small rising in the ground, by the chain.

Having set up whites or marks at the places betwixt which you are to measure, go to the rising ground with your assistant; there, at some distance from each other, move backwards or forwards, till you see him in a right line with your second station, at the same time that he observes you in a line with the first station, there set the station staff or some conspicuous mark. Measure from your first station to this mark, and from it to the end of the line, as before.

P R O.

P R O B L E M IV.

To measure a right line when there happens to be some obstruction in the way, such as water, wood, &c. one end of which you can pass by; with the chain.

Let A F, (Fig. 6th. Plate 4th.) be the required line, into which comes the end of the pool G. Measure from A to B, suppose 4 chains, and there raise the perpendicular B C, thus, set your off-set staff at B and measure back from B to *a*, 40 links, and there stick down an arrow, let one end of the chain be kept fast at *a*, and the 80th link at B; while you take the end of the 50th link and stretch it so as the two parts *b a*, and *b B*, may be equally tight, and set down an arrow at *b*, Then is B *a*, 40, B *b*, 30, and *a b*, 50. And therefore the square of B *a*, added to the square

square of Bb , is equal to the square of ab ; by the 47th Prop. of EUCLID'S 1st book of elements, and therefore aBb is a right angle. Continue this line Bb as far as is necessary to carry you by the pool as to C , measure BC , suppose one chain, then raise the perpendicular CD , in the same manner you did BC , and measure CD , suppose 2 chains, and raise the perpendicular DE , and make it equal to BC at E , raise the perpendicular EF , measure EF , suppose 1,58. Then AB added to CD , added to EF is equal to the whole line AF , 7,58, chains, the same as if it had been measured through the pool from B to C .

Note, This method of raising perpendiculars with the chain is troublesome; but

may be occasionally used, when you have no other instrument at hand. How it's done by other instruments will be shewn hereafter.

SECTION

SECTION V.

*Containing the description and use of the
Protractor and Plotting-Scale.*

HAVING explained the instruments necessary for taking the dimensions, either for the contents or the plan of any inclosure, or other parcel or parcels of land. But perhaps some will say I ought to have explained the Theodolite and Plain Table; to which I answer, such as have those instruments need no explication, and such as have them not, I would recommend the foregoing sextant as preferable to either; as I hope to make appear in the following parts of this work. I

shall therefore proceed to the explanation of the instruments necessary for planning,

And first of the PROTRACTOR,

This instrument is generally made of brass, of various sizes from 4 to 12 inches diameter, (about 8 is a good size) and in a semicircular form, see Fig. 7th. whose limb is divided into 180 equal parts called degrees, and these subdivided again, if the size of the instrument will admit of it; these degrees are numbered both ways by setting figures at each 10th degree, on the outer and inner circles, the divisions on the limb serving equally for both.

There are some protractors, now made with a Vernier's scale, such as described in the 3d Section; where we treated of the sextant. And some there are with diagonals, but these instruments are of too small a radius

SEC. V. OF THE PROTRACTOR. 71

a radius to admit of these latter divisions to be true. But in short we have no need of these very great niceties, as we do not plan for the contents; as other authors have done.

To make or lay down an angle of any given number of degrees by the protractor.

Lay the center of the protractor to the angular point, and bring the fiducial edge (or the edge passing through the center) close to the given line, then from that end of the protractor which cuts this line, count the given number of degrees and minutes in the limb, and there make a point; a line drawn through this point will form the angle required.

Example. Suppose it be required to make an angle at the point C, Fig. 7th, with the line C A, of 45 degrees; lay the center of

the protractor to the angular point C, so that the fiducial edge of the diameter lies all the way close to the given line A C; then from this line count 45 degrees, and there make a mark or prick with your protracting pin, and through this mark draw the line C D, which forms the angle required: this line also forms, with the line C B, an angle of 135 degrees, as is shewn by the inner scale of numbers. In like manner an angle at C may be made with the line C B, for instance of $34^{\circ}, 40'$, having laid the edge of the protractor close to the line C B, and center to C, count 34° on the innermost scale, and for the 40 minutes, estimate as near as you can two thirds of a degree more, and there make a prick or point; a line C E, drawn through this point, makes the angle B C E, $34^{\circ}, 40'$; whence its supplement or angle
A C E

SEC. V OF THE PROTRACTOR. 73

A C E, is $145^{\circ}, 20'$, as is shewn by the outermost scale.

How to find the number of degrees contained in an angle already made is self-evident ; for having adjusted the protractor to the center, and one of the legs of the given angle, the other leg (produced if need be) will cut the limb of the protractor in the degrees which the angle contains ; suppose the angle A C D, Fig. 7, be required ; set the center of the protractor to the angular point C, and the edge of its diameter to the line A C ; then will the line C D cut the limb of the protractor in 45 degrees, (counting from the line A C) the angle sought. And in the same manner the angle B C D will be found 135 degrees, counting from the line B C.

Note,

ing chains is half an inch. These I call the quarter and half inch running scales.

Note, The divisions marked o, are drawn quite across the ruler and exactly perpendicular to its edges.

On the under or backside which is flat, and marked B, is a diagonal scale of half an inch to a chain; this scale contains eleven parallel lines, drawn exactly at equal distances, nearly the length of the ruler; and these are divided into equal parts of half an inch each, by perpendicular lines drawn quite across them, and commonly numbered at every division: the first division is subdivided into ten equal parts, both in the upper and lower parallels, and oblique lines drawn from the beginning below, to the first subdivision above, and from the first subdivision below, to the second subdivision above, &c.

divid-

SEC. V. THE PLOTTING SCALE. 77

dividing the first half inch into one hundred equal parts : if therefore the larger divisions be accounted units or chains, the first subdivisions will be tenth parts of an unit, or of a chain, viz. ten links ; and the second subdivisions made by the diagonals on the parallels will be hundredth parts of an unit, or of a chain, and consequently each one link. Again, if the larger divisions be reckoned tens, the first subdivisions will be units, and the second subdivisions tenth parts ; and if the larger divisions be accounted hundredths, the first subdivisions will be tens, and the second units ; and so on.

The

The use of the above SCALES.

1st. *Let it be required to lay down the line A F, (Fig. 6.) of 7,58 chains by the quarter running scale, (see prob. 4. sec. 4.)*

Draw an occult line with the point of your compasses, in the position you would have the line A F, at one end make a prick to represent the point A, lay the edge of the quarter scale to this line in such manner that the division marked 0, may fall exactly on the prick or point A : then with your protracting pin make a prick at the division, representing 4 chains, and it gives the point B : make another prick at 6 chains, and it gives the point E ; also make a prick very near the 6th subdivision between the 7th and 8th division, and it gives the point F, 7 chains
and

SEC. V. THE PLOTTING SCALES. 79

and 58 links: the 6th subdivision being 60 links, because each subdivision is 10 links as before observed.

If you turn your scale so as the point *o* may fall on the point *B*, and the line drawn from *o* across the scale may coincide or fall exactly on the line *A B*, the edge of the scale will be perpendicular to the line *A B*; then against one chain make a prick, and it gives the point *C*: do the same at *E*, and it gives the point *D*; join the points *A B*, *B C*, *C D*, *D E*, and *E F*, and they will represent the lines measured with the chain in such proportion, as a quarter of an inch is to one chain. And exactly in the same manner may the above, or any other line be laid down by the half-inch running scale.

2°. *To measure a line that is already laid down by the running scale.*

Apply the edge of the scale to the line required to be measured, so that the beginning or division marked 0 may fall exactly on one end of the line; and against the other end you have the chains and links required.

Example. Let it be required to measure the line A F, Fig. 6th. Apply the edge of the scale to the line so that the 0 may fall exactly on the point A, and against F is 7,58 that is 7 chains and 58 links required.

3°. *To lay down a line of 7,58 chains by the diagonal scale.*

Having drawn an occult line as before directed; set one foot of the compasses in
the

SEC. V. THE PLOTTING SCALES. 81

the division marked 7, which in this case stands for 7 chains; and carry it in that perpendicular until you come to the 8th parallel, counting upwards: extend the other point of the compasses to the intersection of the same (8th) parallel, with the 5th diagonal; and this extent will be 7.58 chains, which prick down on the occult line, with each point of the compasses: connect these points and it will be the line required.

Note, The line laid down by this scale will be double in length to that of Fig. 6, by reason this scale is double of the scale that figure was laid down by. Nevertheless you may by this scale lay down any line in proportion of a quarter of an inch to a chain, by first halving the number of chains and links in the proposed line, and taking those halves from this scale. Suppose

G

pose

pose for example, the above line of 7.58 chains ; the half of which is 3.79 chains : set one foot of the compasses where the 3d division or chain cuts the 9th parallel, or topmost but one, extend the other point to the intersection of the same parallel with the 7th diagonal, and this extent will be equal to 7.58 chains on the quarter scale, and may be laid down with much more exactness by this than by a quarter scale.

4^o. *To measure a right line that is already laid down, by the diagonal scale.*

Take the extent of the line in the compasses ; place one foot on the first parallel, at such of the divisions or chains, as will bring the other foot among the diagonals ; move both the feet upwards till one of them fall into the point where the diagonal from the nearest tenth cuts the same parallel.

SEC. V. THE PLOTTING SCALES. 83

parallel as the other foot cuts in the perpendicular; then one foot shews the chains, and the other the hundredth parts of a chain, or number of odd links.

Example, Suppose one foot of the compasses standing in the point of intersection of the 8th diagonal with the 4th parallel, at the same time as the other foot is in the intersection of the same parallel with the 12th chain, or perpendicular division marked 12; then one foot shews 12 chains, and the other 84 links, or 12,84 chains the length required.

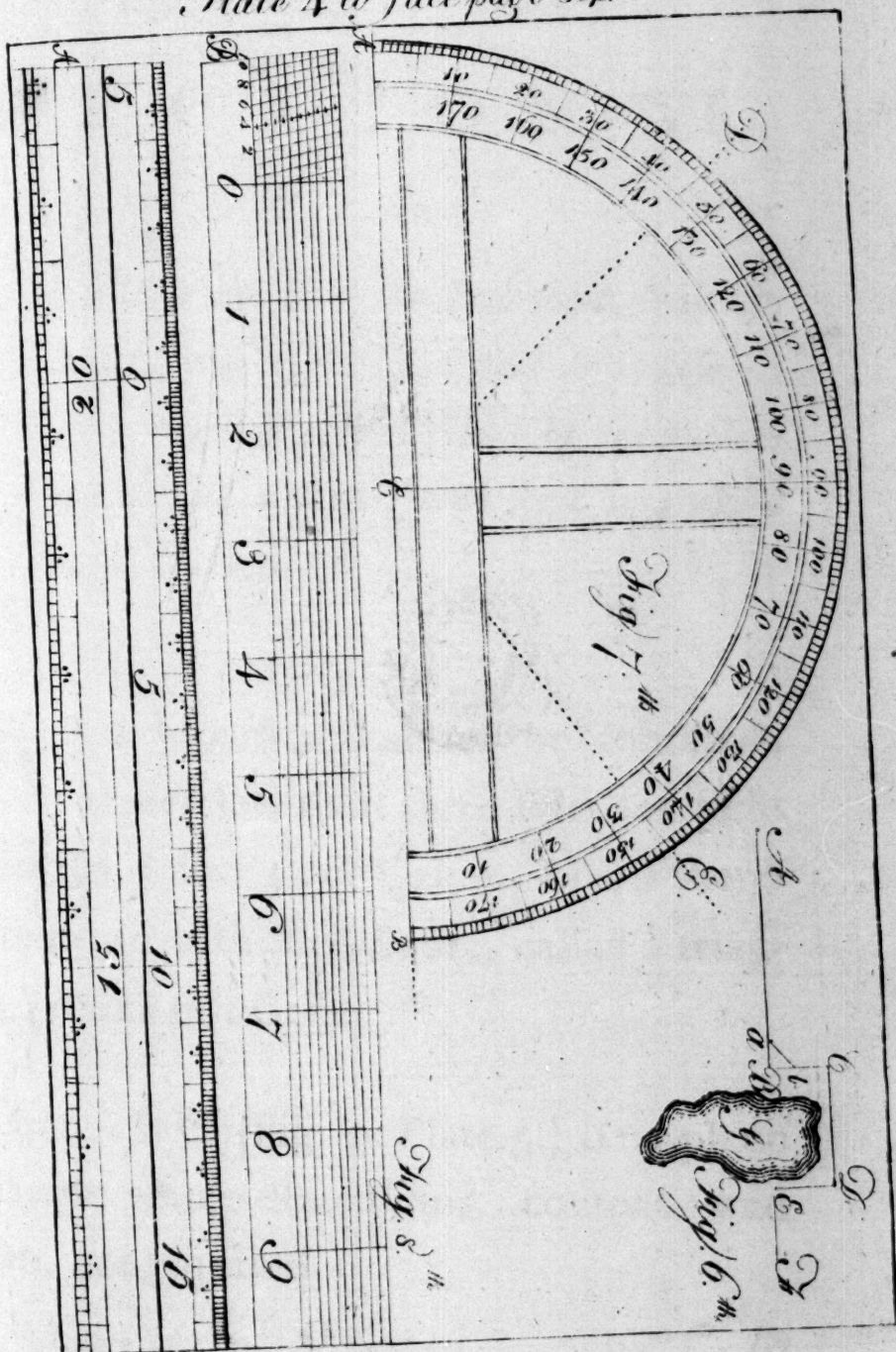
N. B. If the line has been laid down by a quarter scale, and is measured by the half-inch one; the line thus found must be doubled: thus the above line by the quarter scale will be 25,68 chains.

Sec. V. THE FLOTTING SCALLES.

parallel as the other foot, and the same
condition; when one foot is
longer, and the other the shorter, the
condition of the number of odd links
is changed. Suppose one foot of the
same number in the point of intersection
of the 8th diagonal with the 4th parallel,
as the other, and as the other foot is in the
intersection of the same parallel with the
4th diagonal, or perpendicular division
marked 12; when one foot is 12
links, and the other 8 links, or 12
links, the length required.

Ex. II. If the line has been laid down
by a quarter scale, and is marked by the
10th inch one; the line thus found is
doubled; then the above rule by the
quarter scale will be as before.

Plate 4 to face page 84.





S E C T I O N VI.

Shewing how to take the dimensions, cast up, and plot, any right lined field or inclosure; whether regular or irregular, by various instruments and methods, with some very useful remarks thereon.

AS a superficies cannot be contained under less than three sides, or right lines (by defin. 3d of Geom.) I shall therefore begin with such figure, called a triangle (defin. 6th Geom.)

Let A B C (Fig. 9, Plate 5.) be a plane triangle whose dimensions, content, and plan, are required.

G 3

1°. By

1°. *By the Chain only,*

Begin at A, and measure the side A B, according to the directions given under the use of the chain ; suppose 11,41 chains, that is 11 chains and 41 links ; which note in your field-book or on a piece of paper ; then measure B C 10,56, and C A 7,11 chains, which also note down in your field-book or paper, and the dimensions are finished.

Now the area or content is found from these dimensions by the following problem.

P R O.

P R O B L E M I.

Having the three sides of a plane triangle, to find the area.

R U L E.

Add the three sides together, and take half their sum; then from the half sum subtract each particular side: multiply the half sum by the first difference, this product by the second, and that by the third difference; the square root of this last product will be the area.

G 4

EXAMPLE

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E X A M P L E.

sides } A B---11,41
 } B C---10,56
 } C A--- 7,11

2)29,08 sum

14,54 half sum

14,54 half sum

11,41

14,54

10,56

14,54

7,11

3,13---1st diff.

3,98

2d diff.

7,43

3d diff.

half sum

14,54

1st diff.

3,13

4362

1454

4362

45,5102

2d diff.

3,98

3640816

4095918

1365306

181,130596

3d diff.

7,43

543391788

724522384

1267914172

1345,80032828

the last product

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3) 1345,80032828 (36,6851 the square root.

9

66) 445
396

726) 4980
4356

7328) 62403
58624

73365) 377928
366825

733701) 1110328
733701

376627

this 36,6851 is the area in square chains ;
and as 10 square chains (see Table 1, fol-
lowing) are contained in an acre ; if
36,6851 be divided by 10 the quotient
3,66851 will be acres and decimal parts of
an acre ; which parts multiplied by 4,
(the roods in an acre) the product will be
roods, and decimal parts of a rood ; and
these parts multiplied by 40, gives perches
and

and parts of a perch; which is the least denomination generally set down by surveyors.

Thus,

	1(0)3)6,6851	area in chains
Acres	<u>3,66851</u>	
	4	
Roods	<u>2,67404</u>	
	40	
Perches	<u>26,96160</u>	

so the area or content is 3 acres, 2 roods, and 26,96, &c. or 27 perches.

N. B. It is usual for authors to set down the dimensions in links; that is, without any separating point between the chains and links; and so find the area in square links; which divided by 100000 the square links in an acre (see Table 1) the quotient is acres and decimal parts of an acre; which are reduced into roods and perches as before.

But

But instead of actually dividing by 10 as above, or by 100000; point off to the right hand one figure for each cypher, and what remains to the left (if any) will be acres; that is, in the former case (which I always make use of) separate by a line or dash with the pen, one figure more than the decimals; and the figures on the left of the dash are acres, and those to the right decimals; as 36,6851 square chains become 3¹6,6851 acres, &c. and 366851 square links become 3,66851 acres, &c, by pointing of five figures for the five cyphers in the latter case.

Remark. And thus the surveyor is by a great deal of labour, arrived at the true content of this little triangle; which method is nevertheless recommended by some late authors, as the only true method of finding the area of a triangle by the chain only:

only : and also made use of to confront the truth of other methods commonly made use of by the theodolite or plain table ; pretending to demonstrate that no polygonal field can be truly measured by either of these instruments.

But I think it is much easier to demonstrate that any such field may be truly measured by either of those instruments, though not in the common way of using them, as will be shewn by and by. Therefore the poor instruments are condemned without occasion, the fault being in the surveyor ; or rather in the authors who have heretofore taught the use of them.

The

The plan of this triangular field is easily obtained from the 9th prob. of Geometry, thus, (see Fig. 9, Plate 5.)

From the point A draw an occult line, to which apply the edge of your plotting scale, so that the beginning may coincide with the point A, then against 11,41 chains make a prick with your protracting pin as at B.

Take 10,56 chains in your compasses from the same scale made use of for A B, and with one foot on B describe an occult arch; also with 7,11 chains and one foot on A describe another arch so as to intersect the former in the point C; draw the lines A B, B C, and A C, and they will form the plot required.

Before I proceed to the other methods, it may be proper to add the following useful tables.

TABLE

T A B L E I.

Of Square Measure.

Links	Yards	Poles	Chains	Roods	Acres
20,657	1				
625	30,25	1			
10000	484	16	1		
25000	1210	40	2,5	1	
100000	4840	160	10	4	1

T A B L E II.

Of Long Measure.

Inches	Links	Yards	Poles	Chains	Furlongs	Miles
7,92	1					
36	4,545	1				
198	25	5,5	1			
792	100	22	4	1		
7920	1000	220	40	10	1	
63360	8000	1760	320	80	8	1

T A B L E III.
Of Angles and their Factors.

Ang.	Factor.	Ang.	Ang.	Factor.	Ang.
43	,3409	137	67	,4602	113
44	,3473	136	68	,4636	112
45	,3535	135	69	,4668	111
46	,3596	134	70	,4698	110
47	,3656	133	71	,4727	109
48	,3715	132	72	,4755	108
49	,3773	131	73	,4781	107
50	,3830	130	74	,4806	106
51	,3880	129	75	,4829	105
52	,3940	128	76	,4851	104
53	,3993	127	77	,4872	103
54	,4045	126	78	,4890	102
55	,4095	125	79	,4908	101
56	,4145	124	80	,4924	100
57	,4193	123	81	,4938	99
58	,4240	122	82	,4951	98
59	,4285	121	83	,4962	97
60	,4330	120	84	,4972	96
61	,4373	119	85	,4981	95
62	,4414	118	86	,4987	94
63	,4455	117	87	,4993	93
64	,4494	116	88	,4997	92
65	,4531	115	89	,4999	91
66	,4567	114	90	,5000	90

T A B L E IV.

Of Chords to the radius of 100 Links.

An.	Chord	An.	Chord	An.	Chord	An.	Chord
43	73,3	67	110,4	91	142,6	115	168,7
44	74,9	68	111,8	92	143,8	116	169,6
45	76,5	69	113,3	93	145	117	170,5
46	78,2	70	114,7	94	146,2	118	171,4
47	79,7	71	116,1	95	147,4	119	172,3
48	81,3	72	117,5	96	148,6	120	173,2
49	82,9	73	119	97	149,8	121	174,1
50	84,5	74	120,4	98	151	122	174,9
51	86,1	75	121,8	99	152,1	123	175,7
52	87,7	76	123,1	100	153,2	124	176,6
53	89,2	77	124,5	101	154,3	125	177,4
54	90,8	78	125,9	102	155,4	126	178,2
55	92,3	79	127,2	103	156,5	127	179
56	93,9	80	128,5	104	157,6	128	179,8
57	95,4	81	129,9	105	158,7	129	180,5
58	97,0	82	131,2	106	159,7	130	181,3
59	98,5	83	132,5	107	160,8	131	182
60	100	84	133,8	108	161,8	132	182,7
61	101,5	85	135,1	109	162,8	133	183,4
62	103	86	136,4	110	163,8	134	184
63	104,4	87	137,7	111	164,8	135	184,7
64	106	88	139	112	165,8	136	185,4
65	107,4	89	140,2	113	166,8	137	186,1
66	108,9	90	141,4	114	167,7	138	186,7

TABLE V.		
<i>Of Roods and Perches to chains & decimals.</i>		
Chains, &c.	R.	P.
,0625	0	1
,1250	0	2
,1875	0	3
,2500	0	4
,3125	0	5
,3750	0	6
,4375	0	7
,5000	0	8
,5625	0	9
,6250	0	10
,6875	0	11
,7500	0	12
,8125	0	13
,8750	0	14
,9375	0	15
1,0000	0	16
2,0000	0	32
3,0000	1	08
4,0000	1	24
5,0000	2	00
6,0000	2	16
7,0000	2	32
8,0000	3	08
9,0000	3	24

The foregoing TABLES explained.

The first is a table of square measure, and may be read thus: in one acre are 4 square roods, or 10 square chains, or 160 square poles or perches, or 4840 square yards, or 100000 square links. Also in 1 rood are 2,5 square chains, or 40 square poles or perches, &c.

The second is a table of long-measure, and read thus; 1 mile contains 8 furlongs, or 80 chains, or 320 poles, or 1760 yards, or 8000 links, or 63360 inches, &c.

The third is a table of Factors answering any given angle between 43 and 137 degrees, which are all I have found necessary in my practice of surveying.

These

These factors are the half of the natural sines of the angles they stand against, reduced to the radius 1.

If the given angle is less than 90 degrees, it is found in the first or fourth column, and the factors against it in the second or fifth column: but if the angle be more than 90° , it is found in the third or sixth column, and the factors in the same as before: observing that each factor has two angles, except that for 90° .

Suppose the factor for an angle of 82 degrees be required; find 82° in the fourth column, and against it in the fifth is, 4951 the factor sought. Again it is required to find the factor for an angle of 98 degrees, find 98° in the sixth column, and against it in the fifth is ,4951, the same as before for 82° . In the same man-

ner may the factor for any other angle be found.

The fourth is a table of chords, or subtenses of the angles they stand against, to the radius of one chain or 100 links; or these chords are the third side of an isosceles triangle, the two other being each one chain, (see Diff. 4th Geom.) therefore if the chord or subtense be given, the corresponding angle is given also by this table: thus suppose the chord be 119 links, what is the corresponding angle: seek 119 in one of the columns marked chord, and against it in the next column on the left hand is 73 degrees, the angle required.

The fifth is a small table for the ready finding the roods and perches answering any number of square chains and decimals
of

of a chain less than ten. Suppose there was given 36,6851 (see the example to prob. 1. sect. 6.) having cut off the 3 for acres, there remains 6,6851 square chains: find the integer 6 in the left hand column towards the bottom, and against it in the other columns under R. and P. is 2 roods and 16 perches: find also the decimals ,6851 or the nearest to them in the left hand column, and against them is 11 perches, which, added to the 2,16, makes 2 roods 27 perches, for the answer: and so for any other.

This saves the trouble of multiplying by 4 and by 40, and will be found very useful and expeditious in practice.

Having explained the tables, I shall now proceed to the other methods of surveying a triangular field.

2d. *To survey a triangular field by the chain and cross.*

This is by several authors (I think improperly) called surveying with the chain only. (See Wild's practical surveyor, page 45; also Leadbetter's young mathematicians companion, page 345)

The cross * is only a small bit of hard wood, about 3 or 4 inches diameter fixed on the head of a staff about your own height, and having two flits fawn in it with a fine saw crossing each other exactly at right angles.

Having set up whites at the corners of the field, (Fig. 9, Plate 5.) begin at one of them, suppose A, and measure on in a right line towards B, till you think you are come near to the point D, where a
perpen-

* Some have them made with brass sights.

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perpendicular will fall from the angle C, then letting the chain lie in the line A B, stick down the cross close to the side of the chain, and so as to see through one of the slits, the white at A or B; then look through the other slit for the angle C, and observe whether it falls short, cuts, or falls beyond the said angle C; if it falls short, move forwards; if beyond, move backwards; till by a few trials, you find the exact point D, where the cross being stuck down, you can through one of the slits see the white B, and through the other the point or white C: observe how many chains and links this point D is from A, suppose 3,03, then let your assistant, leaving an arrow at the end of the fourth chain, turn towards C, and you leaving your arrows with the cross at D, measure the perpendicular D C: and having en-

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tered the same in your field-book, suppose
6,43 ; give your assistant the arrows, and
return and measure out the base or line
A B, 11,41 chains, which also enter in
your field-book, and the dimensions are
finished, and the area found by the pro-
blem following.

P R O B L E M II.

*The base and perpendicular of a right lined tri-
angle being given, to find the area.*

R U L E.

Multiply the base by half the perpendi-
cular, or half the base multiplied by the
whole perpendicular, gives the area ; or
multiply the base and perpendicular toge-
ther, and half that product will be the
area.

Example

SEC. V. CHAIN AND CROSS. 103

Example of the foregoing dimensions :

Base	11,41
half the perpendicular	3,215
	5705
	1141
	2282
	3423
product square chains	3)6,68315
	4
	2)673260
	40
	26)930400

hence the content is 3 acres, 2 roods, and 26,93, &c. or rather 27 perches, the same as found by the first method.

Remark. Thus you see the area of this triangle is truly obtained with much more ease and expedition both in the dimensions and computations, than by the first method ; and therefore is much preferable in practice.

The

The sextant may with advantage be a substitute to the cross, as a perpendicular may be let fall from any required point ; with much more ease, expedition and exactness, by that instrument, than by any other ever yet invented : and for this purpose there is a small hole in the outer edge of the sextant ; into which you may put a peg, and it will stop the index exactly at 90 degrees : then suppose the point D, in the last example, where the perpendicular falls from C, is required: being come, as you think, near the point D, set the index to 0, (the instrument being already adjusted) and looking through the sights at the mark at B, bring its image towards C, by sliding the index till it stops at the peg or 90th degree ; then observe whether the said image falls short, coincides, or falls beyond C ; if it falls short,
move

SEC. VI. CHAIN AND CROSS. 107

move gently forwards along the chain line towards B, if it falls beyond, move backwards towards A, keeping your eye sometimes on the image, and sometimes on the white at C, and you will find the image approach as it were, the point C, and thus move on till the image and white do exactly coincide, which will be at the point D, required: for the index being at 90° and the image at B, brought to coincide with the point C: its plain the angle BDC must be a right one or 90 degrees: In the same manner may the image of the mark at C be brought to coincide with the mark at A.

The plan is had from the foregoing dimensions thus; from your diagonal scale, make AB equal 11,41 chains, and AD 3,03 chains; on the point D erect the perpen-

perpendicular D C, (by prob. 1st, Geom.) and make it equal 6,43 chains, then draw A C, and B C, and its done. (See fig. 9.)

This may be planned more expeditiously by the running scale; see its explanation and use in Section 5.

3^o. Method by Chain and Sextant.

The 3^d method I shall here make use of to measure the triangular field, fig. 9, I believe is entirely new, as I never saw it in any author, nor practised by any surveyor, till I used it, and taught it my pupils and others.

Having set up whites at the corners, begin at one of them as A, and measure the side A C 7,11 chains, which write in your field book; then standing at the angle C, find with your sextant the angle A C B, by bringing the image of the white
at

SEC. VI. CHAIN AND SEXTANT. 109

at A to coincide with the other at B; then will the index shew the degrees and minutes of the angle to be $77^{\circ}, 44'$ (see the use of the sextant in taking horizontal angles, sect. 3d) set down this angle in your field book, and measure the side C B 10,56 chains, which set down in your field book, and the dimensions are finished.

The area is found by

P R O B L E M IV.

Having two sides and the including angle, of any right lined triangle given, to find the area.

R U L E.

Multiply the two given sides together and that product multiplied by the factor,
found

found in table 3d, answering the given angle gives the area.

E X A M P L E.

$$\begin{array}{r}
 \text{CB} \text{---} 10,35 \\
 \text{AC} \text{---} 711 \\
 \hline
 1056 \\
 1056 \\
 7392 \\
 \hline
 \text{product of sides} \quad 75,0816 \\
 \\
 \text{product of sides} = 75,0816 \\
 \text{factor to } 77^{\circ}, 44' = ,4885 \\
 \hline
 3754080 \\
 6006528 \\
 6006528 \\
 3003264 \\
 \hline
 \text{square chains} \quad 36,67736160
 \end{array}$$

here is 36 square chains and eight decimal places of figures ; but as four decimals are sufficiently near the truth, it is better to multiply the first product by the factor inverted,

SEC. VI. CHAIN AND SEXTANT. 111

verted, as is commonly taught in decimal arithmetic.

$$\begin{array}{r}
 \text{Thus product fides } 75.0816 \\
 \text{factor inverted } 5884.0 \\
 \hline
 300326 \\
 60065 \\
 6007 \\
 375 \\
 \hline
 36,6773
 \end{array}$$

this reduced is 3 acres, 2 roods, 26,83 perches, the true content as before.

In table the 3d, the factors are set only to even degrees, but are easily obtained for any odd minutes, by taking the difference between the two nearest degrees; and adding 1-4, 1-2, 3-4ths, &c. of this difference, as the minutes may require; to the factor for the given degree. Thus in the above example, the difference between the factors for 77° and 78° , is 18; now 44' is 3-4ths of a degree, therefore add
3-4ths of

3-4ths of 18, viz. 13 to ,4872 the factor for 77° , and it gives ,4885 for $77^\circ, 44'$. Or by proportion thus, as 1° or $60'$: 18 diff. : : $44'$: 13 diff. as above ; but in general the difference may be had by inspection.

To plot this field from the last dimensions,

From the scale you think most convenient lay down the line $AC = 7,11$ chains, then lay your protractor, with the diameter on the line AC , and center on the point C , and prick off the angle ACB , equal to $77^\circ, 44'$, that is, $77^\circ 3-4$ nearly, make $CB = 10,56$ chains, and draw AB , and your plot is finished.

This may also be plotted by help of a line of chords, instead of the protractor ; for having laid down the line $AC 7,11$; with the radius or chord of 60° in your compasses, on the point C describe the arch

arch $a b$; then take in your compasses the chord of the angle $A C B = 77 \frac{3}{4}$ degrees, and set from a to b , and through b , draw $C B$ making it equal 10,46, and draw $A B$, and its done.

Remark. Several late authors have proved that neither the Plain Table nor Theodolite, are instruments sufficiently correct for measuring land in the common way; they suppose (and I believe very justly) that an angle cannot be taken nearer than ten minutes of the truth; with the new improved theodolites; (see page 17 of WRIGHT's introduction to land measuring, new methodized; and GRAY's introduction to his art of land measuring, page 7.) Now this error of ten minutes, amounts to an error of about one acre in 90, or 5 poles in 3 acres, as proved by the

last mentioned author. And the former surveying a field of about 9 and half acres, with all possible care, two different ways; with the plain table made a difference of 20 poles.

The content found by the plain table will in general be less than the truth, by reason of the papers expanding in the field (especially in damp weather) where the dimensions are laid down. And contracting in a warm dry room, where the contents are generally cast up: for if the lines are now measured again by the same scale they were laid down by; they will be found considerably too short, and consequently the contents too small.

I shall now shew that the foregoing field or any other, may be measured sufficiently exact by either of the just-mentioned instruments applied to this my new method;
for

SEC. VI. AND THEODOLITE. 115

for having measured the side AC, plant your theodolite or plain table at the corner C, and take the angle ACB suppose 78, which is 16 minutes more than the truth, instead of 10, and measure the other side CB 10,56 chains, now with these dimensions find the area by the last problem.

thus the product of the sides 75,0816
factor to 78° inverted = 984,0

300326

60065

6757

area in square chains 36,7148

which reduced, is 3 acres, 2 roods, 27,43 perches: here the error is but half a pole, whilst in the common method it would be upwards of 7 poles.

This I think is sufficient to prove that it's not the instruments, but the method of applying them, that has been the occasion of error. And I shall further observe here, that the nearer the including angle is to 90 degrees, the less will be the error in this way of finding the content.

For supposing the sides the same as before, but the true angle at C to be 90° , then the true content will be 3 acres, 3 roods, 00,65 perches, but instead of 90° , suppose the angle be taken 87° , 30', the content will then be 3 acres, 3 roods, 00,04 perches; here the content is but very little more than half a pole less than the truth, though the angle was supposed two degrees and a half too small. Hence it may be worth while to observe, that in finding the contents this way, we should make use
of

of the sides including the angles that are nearest to 90 degrees.

As some of my readers may be doubtful of what I have here asserted; I thought it would be proper to add the following demonstration of the rule.

Let ABC (fig. 10, plate 5.) be a triangle, two of whose sides are given, viz. AB and BC , and suppose the included angle ABC a right one: on B , as a center with BC , describe the semicircle ICF , and make the angles DBC and ECB equal each other, and draw DE and it will be parallel to AB ; compleat the triangles ADB and AEB , whose areas will be equal, (38 EUCLID. 1.) each having the same base AB , and altitude DG , equal EH . Now supposing BC , equal DB , equal BE , equal radius, (1) then will CB ,
I 3
DG, and

D G, and E H, be each equal the sine of the angles A B C, A B D, and A B E, respectively ; therefore the areas of the triangles A B C, and A B D, are in proportion as the sines of their including angles. (1 EUCLID, 6.) And A B multiplied by B C, multiplied by radius (1), is equal twice the area of the triangle A B C, (41 Euc. 1.) therefore A B multiplied by B D, multiplied by sine angle A B D, is equal twice the area of the triangle A B D. Hence A B multiplied by B C, multiplied by half radius, is equal the area of the triangle A B C ; and consequently A B multiplied by B D, multiplied by half sine of the angle A B D, is equal the area of the triangle A B D, which was to be proved,

C O R O L.

COROLLARY I.

Seeing the areas of the triangles ABD , and ABE , are equal, though the including angles at B are very unequal, the one being as much under 90 degrees, as the other exceeds it; shews the reason why each factor in table 3d, has (or serves) two angles, every angle and its supplement having the same sine.

COROLLARY II.

When the angles ABD or ABC become nearly equal the angle ABC or 90 degrees, then their sines or perpendiculars DG or EH , become nearly equal to CB , or radius; consequently the area of the triangles ABD or ABE become nearly equal the area of the triangle ABC ;

therefore no great exactness is required in an instrument for taking the angle in such a case, as a degree or two will make no material difference in the areas.

4. *Method by the Chain only.*

The method I am now going to explain is different from the first method, which was also by the chain only; the dimensions and calculations, being here greatly shortened by help of the 4th table.

Let the area of the same triangular field be required; having measured the side A C (as before) 7,11 chains, then flick down your off-set-staff, or an arrow through the handle or bow of your chain at C, let your assistant at the other end of the chain, holding it tight, flick down an arrow exactly in the line or direction C A

as

as at d ; then let him go into the direction CB , and still holding the chain tight, flick down an arrow at the end exactly in the direction CB , as at e , then measure very exactly the chord de 125,5 links, or one chain 25 and a half links, note this in your field book for the chord of the angle C , take up the arrows, and measure the side CB , and the dimensions are finished.

The area is found by prob. 4, exactly the same as in the last method; for finding 125,5 in table 4, in one of the columns marked chord, you have against it in the next column to the left hand, the angle answering: that is the angle at C equal $77^{\circ}, 44'$; therefore multiplying the sides together, and that product by the factor for $77^{\circ}, 44'$, taken out of table 3d, gives the area the same as in the last method

thod and example, and needs not to be here repeated.

The plot is made from the foregoing dimensions as follows : first draw the line $A C$, making it equal 7,11 chains ; then with an extent of one chain in your compasses, and one foot in C , describe the arch $d e$, in which from d , set 1,255 chains to e , that is, make the chord $d e$, equal 1,255 chains, and through e draw $C B$, making it equal 10,56 chains, then join $A B$ and its done.

N. B. If a small scale be used in plotting, the radius $C d$ of the arch $d e$ will be too small to lay the figure down with exactness : therefore to remedy this ; multiply the dimensions for laying down the angle by 10, then will the radius $C d$ be

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be equal 10 chains, and the chord de , equal 12,55 chains, therefore making them equal these dimensions, the angle C may be laid down from a small scale with the utmost exactness.

Remark. This method is very exact but more laborious than the foregoing one ; and indeed is often impracticable, by reason of various obstacles that may happen in the way, as bushes, furz, trees, water, &c. and in very unlevel ground it is not to be trusted to.

However these difficulties may sometimes be overcome by measuring the angle on the outside the fences ; thus in the direction $A C$ make $C f$ equal one chain, and in the direction $B C$, make $C g$ one chain, and measure the chord fg , which will be equal the chord de of the angle

$A C B$

A C B, as measured within the bounders or fences.

Having, I think, fully shewn the most useful and exact methods for practice ; of surveying and plotting a triangular field whose sides are supposed to be straight or right lines ; and also how to find the contents, by the various methods, without having any regard to the plot.

I shall pass by the methods generally taught by authors, of plotting the land by the Theodolite, Circumferentor, Plain Table, &c. and then finding the contents of the plot by scale and compasses ; because I think they are only near estimations ; and so proceed to the second part of this section.

P A R T

PART SECOND,

SECTION VI.

Shewing how to survey, cast up and plot, any right lined, four, five, six, &c. sided piece of land, whether square, parallelogram, trapezium, &c.

LET a survey of the figures 11, 12, 13, &c. plate 5, be required.

1°. If the field contain but four sides, as suppose figure 13 : begin at one of the corners as A, and take the angle B A D either with the sextant, theodolite, plain-table, or chain, but the sextant is best, as being the most portable, expeditious, and exact ; then measure the line or side A B,
take

take the angle $A B C$, and measure the side $B C$; take the angle $B C D$, and measure the side $C D$; take the angle $C D A$, and measure the side $D A$, and the dimensions are finished. Add the four angles together, and if the sum makes 360 degrees or nearly so; you may conclude that the angles are taken right. I say nearly so, for they will not be exactly 360° , but something more, unless the ground be an even plane, either horizontal, or upon an equal slope; and therefore shews the impropriety of taking angles with the modern theodolite, which measures all angles (let the ground be ever so unlevel) on the plane of the horizon, and consequently often makes the contents of lands less than they really are.

And if by prob. 14, Geom. with the four given sides, and one of the angles, you
make

make the plot ; and then measuring the four angles, if they come out each the same as taken in the field, or very nearly so, will prove the sides to be measured right. But if there should be any material difference, some mistake has been committed, and must be corrected. And thus you may always prove the truth, both of the angles and sides.

The content is had by the rule to Prob. 4, of this Section : thus ; Multiply the sides A B and B C together, and that product by the factor, for the included angle at B, gives the area of the triangle A B C, likewise multiply A D and D C together, and the product by the factor for the angle D, gives the area of the triangle A D C, and these two areas added together, gives the whole area of the trapezium A B C D,

in

in square chains, which may be reduced to acres, roods, and perches, as taught in the foregoing examples of the triangle.

In the same manner may the square parallelogram, and rhombus be measured, if such figures should by chance be met with in practice; and therefore I think it unnecessary to add particular rules for each figure; especially as I apprehend it impossible for a surveyor, when he goes into a field, to know whether it be an exact square, parallelogram, &c, or not, till he has taken the dimensions, or at least a part of them.

All the above figures may be measured by any of the other methods, as taught in measuring the triangle: for if the sides and diagonal be measured, which is according to the first method, the content
may

may be found by the rule to Problem 1 of this Section.

If the diagonal A C be measured, and also the two perpendiculars falling thereon from D and B, found by the cross or sextant, as taught by the 2nd method of the triangle ; observing how many chains and links each falls from A, if the plot is required ; otherwise their distance need not be regarded.

The content may be found by rule to Prob. 2^o. of this sect. for a diagonal reduces all four sided figures into two triangles, and therefore may be measured, and cast up as such. But it will be something shorter to add the two perpendiculars together, then take half their sum, which multiplied into the diagonal, gives the area of the whole figure at once in square
K chains,

chains, &c. or half the diagonal into the sum of the perpendiculars gives the area the same.

An example of the field-book, and calculation, to Fig. 13, will make all plain to the meanest capacity.

N. B. This figure is laid down from a scale of eight chains in an inch, to save room.

The

The FIELD-BOOK to Figure 13.

$\angle A =$	$77^{\circ}, 20'$	
	6,80	= 1st side A B
$\angle B =$	$92^{\circ}, 45'$	
	6,00	= 2d side B C
$\angle C =$	$75^{\circ}, 50'$	
	6,20	= 3d side C D
$\angle D =$	$115^{\circ}, 00'$	
	4,80	= 4th side D A

Here the sum of the angles are only $359^{\circ}, 55'$, wanting $5'$ of 360° , but this shews the angles are taken sufficiently exact, for if each angle had erred $5'$, it would not alter the content half a quarter of a perch.

CALCULATION.

$AB = 6,80$	$CD = 6,20$
$BC = 6,00$	$DA = 4,80$
40,8000	49600
Fact. $\angle B$ 4994,0 invert.	248
163200	29,7600
36720	Fact. $\angle D$ 1354 invert.
3672	
163	119040
	14880
20,3755	893
	30
Area triangle CDA =	13,4843
Area triangle ABC + =	20,3755
Area trapezium ABCDA =	33,8598
	4
	1,54392
	40
	21,75680

Whole content is 3 Acres, 1 Rood, 21,75 Perches.

FIELD-

FIELD-BOOK to the diagonal and
perpendiculars.

Perp. D a = 2,88	3,86	= A a
A b =	5,20	Perp. B b = 4,40
	9,30	= Diagonal A C

CALCULATION.

Diagonal 9,30

Perpen. D a = 2,88

Perp. B b = 4,40

half sum of perpendiculars
Multiply by diagonal

2)7,28

3,64

9,30

10920

3276

33,8520

4

1,54080

40

21,63200

Content = 3 acres, 1 rood, 21,63 perches, the same as
before.

K 3

2° If

2°. *If the field contain more than four sides, as figures 14, 15, and 16.*

Having set up whites at the several angles; get an idea in your mind of the largest four-sided figure that can be formed in the field you are going to measure, which are represented in the above figures by the dotted lines A B, B C, C D, D A.

Then beginning at A, take the angle B A D, and measure in a right line towards B, till you come against the angle *f*, there with your sextant let fall the perpendicular *f e*, (as taught in the 2°. method of the triangle) observing at how many chains and links this off set or perpendicular falls from the beginning or point A, which note in your field-book, and measure *e f*, noting it also in your field-book; then continue the measure of the line A B to B: take the angle A B C, and

and measure the line B C; take the angle B C D, and measure the line C D; take the angle C D A, and measure the line D A; observing as you go round to let fall perpendiculars where necessary, and measure them as specified in the line A B.

EXAMPLE to Figure 16, containing seven Sides.

F I E L D - B O O K.

$\angle A =$	$86^{\circ}, 35'$	
$Ae =$	2,08 5,32	$80 = ef.$ $= 1^{st} \text{ side } A B$
$\angle B =$	$88^{\circ}, 25'$ 4,80	$= 2^{d} \text{ side } B C.$
$\angle C =$	$93^{\circ}, 00'$	
$Cg =$	1,20 4,92	$88 = gb$ $= 3^{d} \text{ side } C D$
$\angle D =$	$92^{\circ}, 00'$	
$Di =$	2,00 4,84	$1,08 = ik.$ $= 4^{th} \text{ side } D A$

Note. Observe to set the off-sets to the right or left of your column of angles and chains, according as they fall to the right or left of your chain line in the field.

CALCULATION.

$AB = 5,32$	$CD = 4,92$
$BC = 4,8$	$DA = 4,84$
<u>4256</u>	<u>1968</u>
2128	3936
<u>25,536</u>	<u>1968</u>
factor $\angle B = 8994$ invert.	23,8128
<u>102144</u>	factor $\angle D = 7994$ invert.
22982	<u>95251</u>
2298	21431
204	2143
<u>12,7628 = area</u>	<u>167</u>
	11,8992 = area

$AB =$

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$$\begin{array}{r}
 AB = 5,32 \\
 \frac{1}{2} ef = \underline{.4} \\
 2,128
 \end{array}
 \qquad
 \begin{array}{r}
 CD = 4,92 \\
 \frac{1}{2} gb = \underline{.44} \\
 1968 \\
 1968 \\
 \hline
 2,1648
 \end{array}
 \qquad
 \begin{array}{r}
 DA = 4,84 \\
 \frac{1}{2} ik = \underline{.54} \\
 1936 \\
 2420 \\
 \hline
 2,6136
 \end{array}$$

$$\begin{array}{r}
 1^{\text{st}} \text{ area } A C B = 12,7628 \\
 2^{\text{d}} \text{ area } A C D = 11,8992
 \end{array}$$

$$\begin{array}{r}
 \text{Sum is the area } A B C D A \quad 24,6620 \\
 \text{area triangle } A B f = 2,1280 \\
 \text{area triangle } C D b = 2,1648 \\
 \text{area triangle } D A k = 2,6136
 \end{array}$$

$$\begin{array}{r}
 3)1,5684 \\
 \underline{4} \\
 0)62736 \\
 \underline{40} \\
 25)09440
 \end{array}$$

Hence the whole area is 3 acres, 25 perches.

Now to make a plot of the same.

The four-sided part is easily plotted by Prob. 14, of Geom. Then laying your plotting

plotting scale on the line A B, with o, on the point A, and at 2,08 chains prick of the point e, turn your scale perpendicular to the line A B, with o, on the point e, prick of 80 links, and it gives the point f, draw A f, and f B. Lay your scale on the line C D, with o, on the point C, and at 1,20 chains prick of the point g, turn your scale perpendicular to the line C D, with o on the point g, prick of 88 links, gives the point h, draw C h and h D; and with o, on the point D, at 2,00 chains prick of the point i, make i k equal 1,08 chains, and draw D k and k A, and its done.

And after the same manner may any other field of a greater number of sides be easily measured, cast up and plotted.

But it may not be amiss to observe to the young surveyor, that it may be of use
some-

sometimes, to plot the figure before he casts it up; as it will give him a better idea of the different triangles formed by the off-sets.

Remark. The common methods of measuring and plotting the foregoing figure by the Theodolite, Plain Table, &c. is by going round them, measuring every side, and taking every angle; which in the last figure above-mentioned has seven sides and as many angles; therefore the Theodolite, Plain Table, &c. must be planted down at seven different stations, which takes up a deal of time and trouble. Then before the content can be found, these sides and angles must all be plotted; and it is well known that if the sides and angles are not taken, and also laid down with the utmost exactness; the two last lines will not close,
and

and therefore the content cannot be had so near the truth as it ought. For as the content is found by measuring the figure by the same scale that the sides were laid down by: 'tis plain that if those sides do not exactly close (and they very rarely will when the utmost care is used) the figure of the field is erroneous; and consequently the area must be so too.

But in the method I have just before been explaining, none of these difficulties or uncertainties happen; but the content is had with the utmost certainty. Besides the dimensions are taken with much less time and trouble.

SECTION

SECTION VII.

Shewing how to survey, cast up, and plot, any inclosure or parcel of land, wood, or water, in the form of a circle, oval, or segment of a circle.

Also any irregular figure whose sides or boundaries, are part right and part curved lines.

THE circle and oval are figures very rarely met with in surveying, unless in pleasure grounds, gardens, fountains, &c. therefore I shall touch but lightly on them. But as the segment of a circle occurs more frequent, shall be more full on that subject; and shew an intire new method

thod of finding the area of the segment of a circle, by help of a small table of factors, which I invented and calculated in 1763, and have since that time taught it to many of my pupils.

1^o. Of the CIRCLE.

Let A D B C A, (fig. 17, plate 5.) be a circular inclosure, whose content is required.

Now in order to find the content we must first find and measure the diameter D E, (as the circumference cannot be well measured by the chain) which may be performed several ways. And first, if the circle be small, so that the diameter does not exceed one chain; let your assistant hold one end of the chain at D, whilst you hold the other part stretched near E, then
moving

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moving it a little backwards and forwards till you find the longest line DE , and that will be the diameter.

But if the circle be large, whose diameter consists of several chains; measure any chord line AB , and re-measure half way back, and you'll have the point C , the middle of the chord line AB . On this point C , with your sextant or other instrument, erect the perpendiculars CD , and CE , setting up marks: measure DE and it gives the true diameter.

Also if the circle should contain wood, water, &c. so that you cannot measure a chord line. Then on the outside, measure any tangent line $aD b$, of a convenient length, touching the circumference in the point D , and make $D b$, equal $D a$, and erect the perpendiculars aA , and bB , measure these perpendiculars, which will
be

be equal each other, if your work is well performed; and each equal the versed sine CD , and ab will also be equal the chord line AB , by which the diameter may be found, by dividing the square of half the chord AC , (or aD equal Db), by the versed sine CD , (or $Aa=Bb$), and to the quotient add the versed sine CD , (or Aa) the sum will be the diameter required.

Suppose for example, half the chord, AC be 3,60 chains, and versed sine CD , 1,80 chains; what is the diameter.

Now 3,60 multiplied by 3,60 is equal 12,9600, the square of half the chord; which divided by 1,80 the versed sine gives 7,20 to which add the versed sine 1,80 gives 9,00 for the diameter of the circle required.

Secondly.

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Secondly. Having found the diameter as above, the content will be had by multiplying the square of the diameter by the decimal ,7854 the product will be the area in square chains, which reduce to acres, roods, and perches, as taught in the beginning of the last section.

Thus the square of 9 chains is 81, and this multiplied by ,7854 gives 63,6174 square chains, or 6 acres, 1 rood, 18 perches, for the content of such circle.

Thirdly. The plot is made by describing a circle with 4,50 chains equal the radius or semi-diameter.

2°. Of the *ELLIPSIS* or *OVAL*.

(Fig. 18, plate 5.)

In order to find the content of an oval, we must first measure the greater diameter

L

A B, and

AB , and the lesser one CD ; which if less than a chain's length, may be easily measured as directed for the circle.

But if they contain several chains, it will be necessary to set up marks at the extremities of the longer diameter as near as you can judge, as at A and B , then measure AB ; and re-measure back the half thereof, (viz.) BE , at E , erect EC , and ED perpendicular to AB , measure CD , and the dimensions are done.

But if the oval contains water, wood, &c. so that the dimensions cannot be taken on the inside, proceed thus :

Measure a tangent line aAb , as near a perpendicular to the longer diameter AB , as you can guess : make Ab , equal Aa , and measure bc , and ad , which will be equal if the tangent line be rightly taken ; otherwise it must be corrected till you get
them

them equal or very nearly so: then continue this line both ways till you find (by your sextant) the points F and G, where perpendiculars will fall thereon, from the extremities of the oval D and C; measure F G, and it will be the shorter diameter, equal C D: continue the line G D to H, where a perpendicular falls thereon from the extremity of the oval at B: then measure H G, and it gives the greater diameter, equal A B.

Now having the two diameters given, the content will be found by multiplying the greater diameter by the lesser; the product multiplied by the decimal .7854 gives the area in square chains; or any other measure, the dimensions were taken in.

EXAM.

E X A M P L E.

Having found (by one of the above methods) the greater diameter to be 9 chains, and the lesser 5 chains 40 links, now 9 multiplied by 5,4 makes 4,86 and this multiplied by ,7854 gives 38,17044 square chains for the area, or 3 acres, 3 roods, and 11 perch.

The plot of this or any other oval is made by the 18th Prob. of Geometry, which need not be here repeated.

3°. *Of the Segment of a CIRCLE.*

Though the practical surveyor seldom meets with the circle or oval; yet in almost every days practice, he meets with
the

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the segments of each or of some other curve; and as all the methods hitherto known for finding the area of segments truly, are very intricate and laborious; few surveyors are prevailed on to practice them, but sooner choose to take several perpendiculars from the chord line to the curve; and casting up the spaces between them as parallelograms: but it is manifest this way gives the area too small, and the method is very laborious.

Others to save trouble, increase the height of the segment or versed sine a few links by guess, and so cast it up as a triangle; but this method is very defective, and ought to be abolished.

This in the year 1763 induced me to try if I could not find out a practical method of finding the area of the segment of a circle by having only the chord of the
segment,

segment, and its versed sine, or height of the segment given : these being the most convenient dimensions to be taken in the field.

My attempts proved successful, and had they fallen into the hands of some of our great mathematicians instead of mine ; would in all probability have made their way through all the academies in Europe ere now.

However, I here offer to the public the result of my labours, not only for the benefit of the surveyors, but also of those who employ them ; well knowing many erroneous contents are given in, for want of such a practical method of finding the area of inclosures, part of whose fences or boundaries are curve lines.

A NEW

A N E W T A B L E.

For finding the area of circular segments.

Quot.	Factor.
,0	,66666
,05	,69796
,1	,67186
,15	,67853
,2	,68720
,25	,69856
,3	,71213
,35	,72785
,4	,74546
,45	,76524
,5	,78524

The use of the above T A B L E.

R U L E.

Divide the versed sine of the segment of
any circle, by the chord of the same seg-

L 4

ment ;

ment; and find the quotient in the first column of the table under Quot. and against it in the column marked Factor, you have a number, by which, if you multiply the product of the chord by the versed sine; gives the area of the segment required.

If the quotient cannot be found exactly in the table; take the factor answering to the number nearest to the said quotient; which will generally be sufficiently exact for the purpose of surveying.

But when more exactness is required, (as in gauging, &c.) take out the factors answering the two nearest numbers, one greater, the other less; and take their difference, and also the difference between the quotient and the nearest number to it in the first column: then say by proportion, as the difference between two quotients

tients in the table, is to the difference between their corresponding factors, so is the difference between your new found quotient, and the nearest in the table, to a fourth number: which fourth number must be added, or subtracted to or from the factor corresponding to the nearest quotient; according as the new quotient is greater or less than the nearest in the table.

E X A M P L E I.

From WARD'S Young Mathematician's Guide, page 410, 8th edit. Where the chord of the segment is 24,98, and the versed sine 6; then 6 divided by 24,98 quotes ,24; and the nearest quotient in the table is ,25, whose factor is ,6985: then 24,98 multiplied by 6 gives 149,88
for

for the product, and this multiplied by the factor ,6985 gives 104,692 for the area of the segment required.

This area though found with the factor answering the nearest quotient in the table, does not differ from WARD, one third of an unit; for the area by him is 104,3544.

But to find it more exact; the diff. of quotients in table, is (always) ,05, and between the new quotient and nearest in table, is ,01; also the difference of factors, for ,2 and ,25 is ,01136. Then say, as ,05 is to ,01 so is ,01136 to ,00227; this subtracted from the factor for ,25, viz. ,69856 leaves ,69829 the factor for ,24 by which multiply 149,88 (the product of the chord by the versed sine) and it gives 104,359 for the area with great exactness.

EXAM-

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E X A M P L E II.

From ROBERTSON'S Menfuration page 161.

Having the chord 48 and the verfed fine 18 to find the area of the ſegment.

chord verfed fine

48) 18,000(,375 new quot,

144 ,35 neareſt in tab.

360
336 ,025 diff.

240

240

...

factor for ,4 is ,74546

ditto for ,35 is ,72785

the diff. - - ,01761

ſay as ,05 : ,025 :: ,01761 : ,00880
to which add the factor for ,35 = ,72785

ſum is the factor for ,375 = ,73665
this multiplied by the product } 864
of chord by the verfed fine } _____

294660

441990

589320

gives the area of ſegment = 636,46560

Note.

Note. WILSON in his book of surveying tells us to multiply the chord by two thirds of the versed sine, to find the area of the segment of a circle: but this rule is very erroneous, unless the versed sine be exceeding small, as may be easily proved by comparing it with the foregoing examples.

Having shewn the general use of the table for finding the area of the segments of circles: I shall proceed to the next article.

3°. To

3^o *To find the area of inclosures or parcels of land, whose fences or bounders are part straight and part curved lines.*

Let it be required to make a survey of such an irregular inclosure, as represented by figure 19, plate 5.

Having looked the land over and set up marks at the most convenient places as A, B, C, D, &c then beginning at A, take the angle B A D, and measure the off-set A *a*, 40 links, then measure on in a right line towards B, till you come to the off-set *b* C, which is 4,14 chains from A, which enter in the middle column of your field-book, and measure the off-set *b c* 72 links, which set on the right hand against the 4,14 chains, because the off-set falls to the right hand of your station, or
chain

chain line ; continue the chain line to *d*, 8,52 chains from A, which enter in your field-book, and against it on the right hand set ,00, because here the chain line touches the fence ; continue your chain line till you are against the widest part of the segment *d B f* as at *e*, 11,24 chains from A, which enter as before directed, and measure the height of the segment or versed side *e f*, ,84 links, which set against the foregoing number, and continue your chain line to B, 14,20 chains, which enter, and against it 00, and write the 1st line, and underneath draw a line ; which finishes the line A B. Then standing at B, take the angle A B C, $91^{\circ}, 05'$, which enter in the middle column under the line just drawn, and proceed to measure the line B C, and having measured along close to the hedge from B to *g*, 2,76 chains,

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chains, enter them in your book and against it 00, being here close to the hedge but going to leave it; measure on to *k*, 6,00 chains, and measure the off-set *h i*, 62 links which enter against the 6,00 chains; and continue the chain line to C, 8,76 chains being the whole line B C, which enter, and against it set 00, being in the corner, write 2d line, and underneath draw a line.

Take the angle B C D, $88^{\circ}, 30'$, measure from C towards D, to *k* 3,60 chains, and the off-set *k l*, 1,12 chains, which being entered as before directed; continue your chain line to *m*, 5,20 chains, measure the off-set *m n* 40 links; and continue your line to *o*, 8,25 chains, measure the off-set *o p*, 62 links, and measure on your chain line to *q*, 10,80 chains; and being here close to the hedge set against it 00, for the
off-set,

off-set, continue the line to D, 12,52 chains, all which being entered, write 3d line.

Take the angle C D A, $102^{\circ}, 20'$, and proceed to measure the line D A, to 14,40 chains (observing the directions given in Sect. 4th. Prob. 4^o. for measuring by the pond) and measure the height of the segment $r f$, 80 links, continue the line to A, 8,80 chains which being entered as before directed, and a line drawn underneath finishes the dimensions. See how all these are entered in the following form of the field-book.

If there be any conspicuous object in the inclosure, at a distance from the chain lines as a barn, tree, &c. whose situation you would like to insert in the plot; it may be easily done thus; suppose the situation
of

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of the tree in fig. 19, be required, at any two convenient places in any of the chain lines, take the angle the tree makes with the said line at those points, as suppose at the points A and B, enter the quantity of these angles in one of the columns of remarks, against the points in the chain line where they were taken ; these angles being protracted, (as will be shewn by and by) will give the true situation of the tree.

Or its situation may also be had by letting fall perpendiculars from the tree, &c. on any two adjacent chain lines, observing at what number of chains and links they fall at ; the intersection of these perpendiculars when protracted will give the exact situation of the tree, or any other object.

M

Here

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Here follows the field-book to figure 19, which is laid down by a scale of 8 chains to an inch.

Remarks, off-sets, &c.	Chain lines and Angles	Remarks, off-sets, &c.
$\angle$ 1st	$78^{\circ}, 10'$	
$\angle 30^{\circ}, 30'$ stat. to tree	000	40
	4,14	72
	8,52	00
	11,24	84 high. segt.
1st line	14,20	00
$\angle$ 2d	$91^{\circ}, 05'$	
$\angle 30^{\circ}, 10'$ back ft. tr.	000	
	2,76	00
	6,00	62
2d line	8,76	00
$\angle$ 3d	$88^{\circ}, 30'$	
	3,60	112
	5,20	40
	8,25	95
	10,80	00
3d line	12,52	
$\angle$ 4th	$102^{\circ}, 20'$	
	4,40	80 higt. segm.
4th line	8,80	

Now the content of this figure may be easily found without the help of the plot ; but as it may be a guide to the unexperienced surveyor, in casting up the off-sets.

I shall therefore proceed to the method of laying down the figure first ; and then shew how to cast up the contents of the same.

By help of Prob. 14th. of Geom. the four station lines are laid down thus ; make A B (from your intended scale) 14,20 chains, then laying the center off your protractor on the point B, and the fiducial line to coincide with A B, (see the use of the protractor, Sect. 5^o.) prick off from the said line A B, the observed angle at B, $91^{\circ}, 05'$, and through this point draw B C, making it 8,76 chains ; then with 12,52 chains in your compasses, and

one foot in C, describe an arch towards D, also with 8,80 chains, and one foot on A, intersect the former arch at D, draw C D and A D, and the right lined part is done. Now lay the edge of your scale to the line A B, with o, exactly on the point A, and the field-book lying open before you, with your protracting pin prick off 4,14. 8,52, and 11,24 chains respectively ; then turning your scale perpendicular to the line A B, first with o, on the point A, prick off to the right 40 links for the point *a*, then move the scale to the next point or prick *b*, 4,14, and prick off the off-set 72 links for the point *c* : at 8,52 the off-set *d* is 00, therefore move the scale to the prick at *e*, 11,24, and prick off the off-set 84 links, and it gives the point *f* the height of the circular segment, join the points *a*, *c*, *d*, with right lines, and the
segment

segment $d f B$, may be drawn through those points with a steady hand; or you may draw it with a pair of compasses by Prob. 17th of Geometry, there being three points given, viz. $d f B$, to find a center.

Having compleated the bounder to AB , lay the scale on the 2d line BC , with o , on the point B , and prick off the lengths $B g$, 2,76 and $B h$ 6,00, then turning the scale perpendicular on the point h , prick off the off-set hi 62 links, and it gives the point i , join $B g$ (in the station line) gi and $i C$, with right lines, and it compleats the bounder to BC . Lay the scale on the line CD , and prick off 3,60, 5,20, 8,25 and 10,80, for the points k, m, o, q , and at these points prick off the several off-sets 1,12, ,40, ,95, ,00, and they give the points l, n, p, q , then join C, l, n, p, q, D , with right lines and they compleat the

bounder to C D. Lastly, lay the scale to the line D A, and prick off 4,40 at r , turn your scale and prick off the height of the segment 80 links, gives the point f , then by help of the three points D f A draw the circular bounder to D A, and the whole is compleated, except finding the situation of the tree at t ; to do which, I see in the field-book that at A, the angle the tree makes with the chain line is $30^{\circ}, 30'$, and the angle at B, that it makes with the same line B A, is $30^{\circ}, 10'$: therefore lay the center of your protractor on the point A, and prick off $30^{\circ}, 30'$ from the line A B; also with the center of the protractor on the point B, prick off the angle $30^{\circ}, 10'$ from the same line A B: now occult lines drawn from A and B, through these points made by pricking of each angle, will intersect each other in the exact place or situation
of

of the tree ; all which will appear plain and easy by a careful inspection of fig. 19.

Now in order to find the area or content of the above figure.

1°. The right lined part or that which is contained with the chain lines, is easily had by the rule to Prob. 4th, Sect. 6th.

Thus ; the two sides including the second angle $91^{\circ}, 05'$ are 14,2 and 8,76 chains ; therefore 8,76 multiplied by 14,2 makes 124,392, and this multiplied by ,4999 the factor for $91^{\circ}, 05'$ gives 62,183 square chains : and 12,52 multiplied by 8,8 (being the other two sides) makes 110,176, and this multiplied by 4,884 the factor for their including angle $102^{\circ}, 20'$, gives 53,809 square chains, which added to those found above, viz. 62,183 the sum 115,992 is the area of the right lined part.

2°. For the off-sets on the first line or $A B$, first find the area of the parallelogram $A b c a$ by multiplying half the sum of $A a$ and $b c$ by the distance $A b$, thus half the sum of 40 and 72 is 56, which multiplied into 4,14 gives 2,3184 the area. Secondly find the area of the triangle $b d c$, by multiplying half $b c$ into the distance $b d$; thus the distance $b d$ is equal 4,14 taken from 8,52 equal 4,38 (see the field-book) which multiplied by half of ,72, viz. ,36 gives 1,5768 the area.

Thirdly for the area of the circular segment $d B f$, which is easily found by the rule given under the use of my new table for finding the area of circular segments; and will be further illustrated by these examples; thus the height $e f$, of the segment ,84 divided by $d B$, the chord 5,68 (found by subtracting 8,52 from 14,20 being

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being the distances where the segment begins and ends) quotes ,14 for quot. against which in the table is ,678, &c. the factor; then the chord multiplied by the height gives 4.7712, and this multiplied by ,678 the factor gives 3,2349 the area of the segment.

Now the areas of the parallelogram, triangle, and segment added together, gives the whole area of the off-sets on the first line equal 7,1301 square chains.

3°. For the area of the off-sets on the second line or B C, which constitute only one triangle, viz. $g C i$, whose area is had by multiplying the distance $g C$ equal 6,00 by half $h i$ equal ,31 equal 1,8600 square chains.

4°. For

4°. For the area to the third line CD , on which there are two triangles and two parallelograms, viz. the triangles Ckl , and qop , the parallelograms $klmn$, and nmp ; now Ck 3,60, multiplied by $\frac{1}{2} kl$,56 gives 2,016 its area: and ,76 the $\frac{1}{2}$ of the sum of kl , and nm , multiplied into 1,60 the distance km gives 1,2160 the area of $klmn$: and $\frac{1}{2}$ the sum of mn , and po , equal ,675 multiplied by 3,05 the distance mo , gives 2,05875 the area of nmp : and lastly oq , equal 2,55 multiplied by $\frac{1}{2} po$, equal 4,75 gives 1,212 the area of qop . Now the sum of these four areas added together gives 6,50275 the whole area to the third line in square chains.

5°. The fourth side DA , contains the segment of a circle only; the whole chain
line

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line being the chord, and r the segments height, or versed sine: now the height ,80 divided by the chord 8,80 quotes ,09 against which, in the table of segments is ,671 the factor; then 8,8 multiplied by ,8, and the product by ,671 gives 4,7238 square chains for the area of the segment being the area to the fourth line or D A.

Now if the area of the right lined part and of the four sides or off-sets be added together, will give the area of the whole figure.

Thus the area of right lined		115,992
Off-sets to A B	—	7,1301
Ditto to B C	—	1,860
Ditto to C D	—	6,5027
Ditto to D A	—	4,7238
whole area in square chains		<u>136,2086</u>

which per table 5th is equal to 13 acres, 2 roods, and 19 perches, for the content of the whole figure,

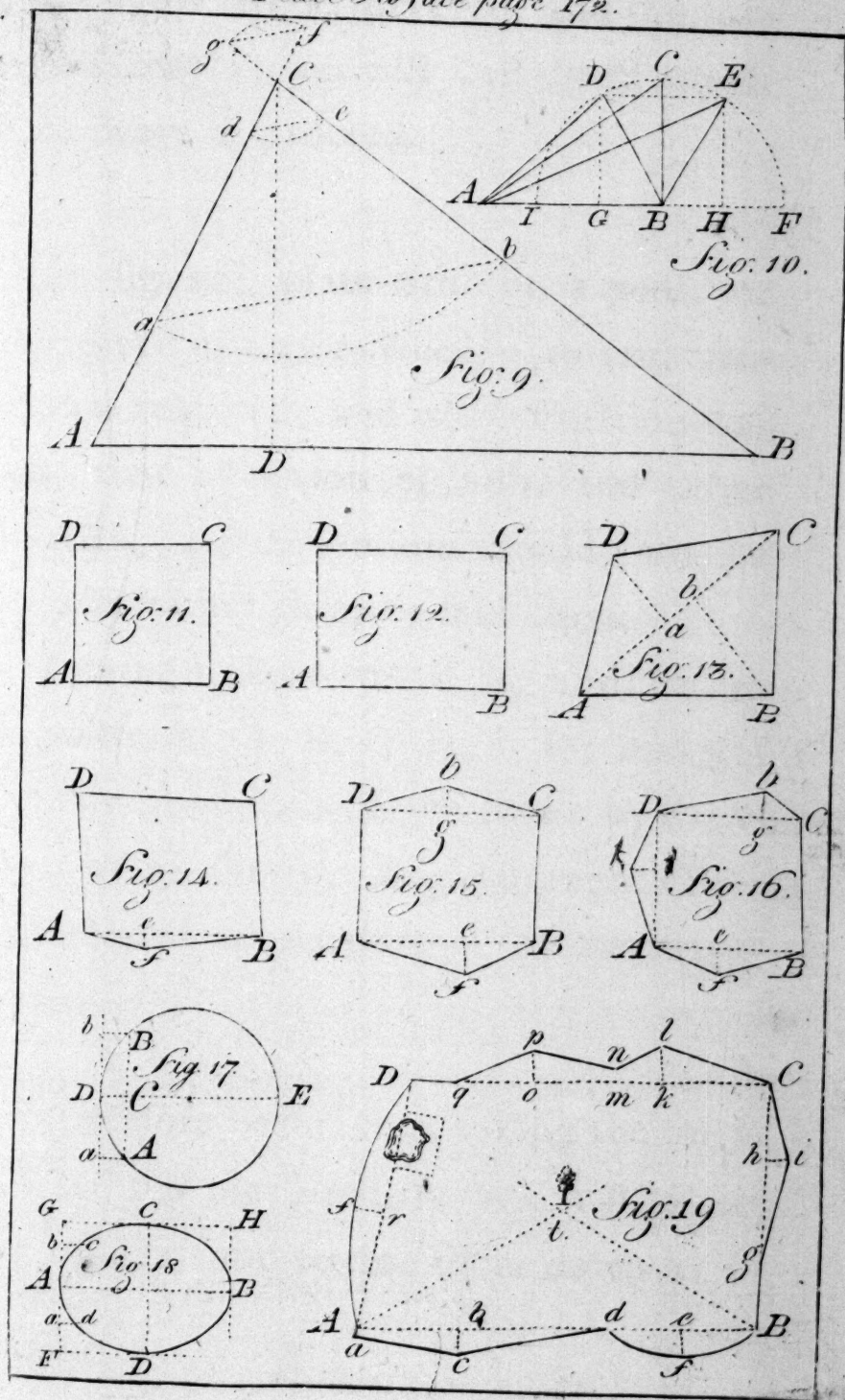
Remark.

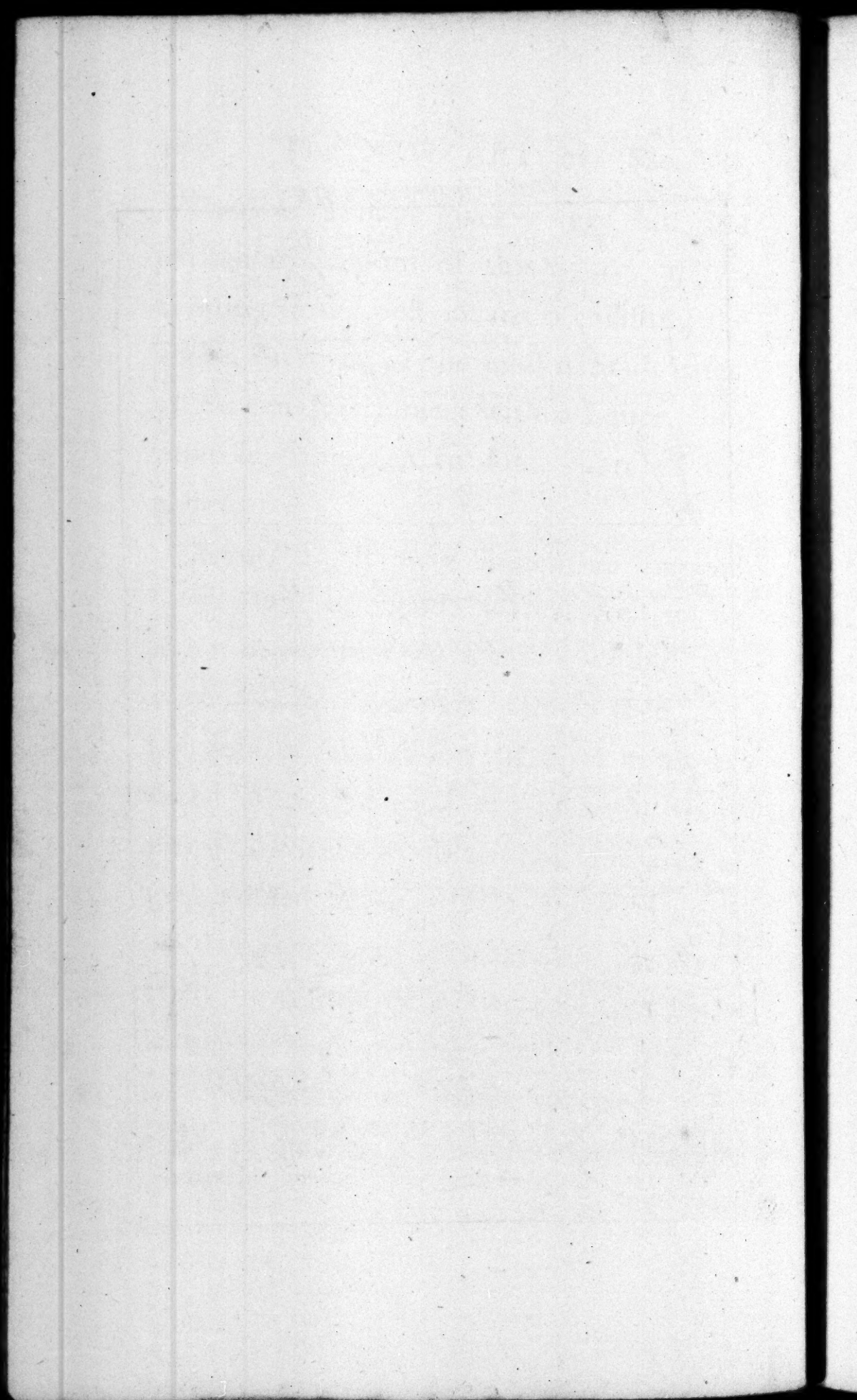
Remark. I have given a very full and particular account of this figure, apprehending it the best means of assisting the young surveyor in the most difficult cases, as he can scarce meet with a figure, but what is contained in some part of the above.

Remark 2. It may sometimes happen on account of wood, water, &c. to be more convenient to measure round the outside of the bounders, and in such case, the area of the off-sets (if any) must be taken from the area of the right lined part; in order to obtain the true area of the inclosed figure.

Also sometimes to measure only three right lines, forming a triangle either within or without the bounders, taking the off-sets thereon as above directed. In either of these cases a careful consideration
of

Plate 5 to face page 172.





of the figures to be surveyed, and the surveyor's own reason will best direct which of the ways to proceed.

Let fig. 20, plate 6th. be a pool, or large piece of water, whose plan and content was required: and whose situation was such, that by reason of hills, and other obstacles, its dimensions could not be taken by either three or four lines.

Having walked round it, and set up the whites at A, B, C, D, E, F; and got a pretty good idea of the figure by drawing a rough sketch, or eye draught of it: I concluded to measure it by two trapeziums.

I therefore began at A taking the angle B A D, and measured the side A B taking the off-sets, and setting them down as in the

the following field-book. I took the angle CBA , and measured the line BC , taking the off-sets, &c. then at C , I took the angles DCB , and DCA , with the utmost care, especially the latter, by repeating it several times, and taking a mean of them all. I then measured CD , and off-sets, and at D took the angles ADC and EDA , repeating the former as mentioned before at C . And so proceeded to measure the lines DE , EF , and FA , taking the angles and off-sets as usual, or as taught before; and being come to A where I began, I took the angle CAD , by the mean of several; then adding the three angles CAD , DCA , and ADC , their sum was 180 degrees or very nearly so, and proves the truth of them. And also the sum of the four angles including each trapezium, made 360 degrees,

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degrees, or very nearly so, and proved the truth of them also: and thus the dimensions were finished.

Now follows the field-book.

Line	Feet	Inches
1	100	00
2	100	00
3	100	00
4	100	00
5	100	00
6	100	00
7	100	00
8	100	00
9	100	00
10	100	00

Line	Feet	Inches
11	100	00
12	100	00
13	100	00
14	100	00
15	100	00
16	100	00
17	100	00
18	100	00
19	100	00
20	100	00

The

The FIELD-BOOK to Figure 20.

Dimensions of — Pool, taken May 30,
1765, by B. T.

$\angle D A B$	$74^{\circ}, 25'$	
60	000	
00	130	
00	285	
210	540	
100	860	
100	1140	
55	1360	
	1405	= 1st line A B
$\angle C B A$	$124^{\circ}, 35'$	
50	070	
00	250	
2d line B C 70	570	$\angle D C A, 53^{\circ}, 00'$

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$\angle DCB$	$93^{\circ},00'$	
00	080	
00	150	
110	425	
140	685	
125	975	
15	1160	
00	1210	
20	1290	
00	1345	
25	1445	
12	1530	
45	1660	= 3d line CD

$\angle ADC$	$68^{\circ},00'$	
		= 4th line DA

$\angle EDA$	$98^{\circ},30'$	
45	000	
180	405	
320	760	
200	1115	
120	1396	
30	1675	
00	1800	= 1st line DE

$\angle FED$	$105^{\circ},20'$	
20	065	
00	140	
25	200	
00	230	= 2d line EF

N

$\angle AFE$	$106^{\circ}, 40'$	
00	000	
00	820	
45	1165	
100	1560	
00	1850	
10	2100	
00	2260	
50	2460	$= 3^{\text{d}} \text{ line } FA$
$\angle DAF$	$49^{\circ}, 30'$	

The fourth line AD , which is common to both the trapeziums, could not be measured with the chain, by reason of the water interfering between A and D , but such as are acquainted with trigonometry, will easily calculate it. Thus, as the sine of angle A is to the side CD , so is the sine of angle C to the side AD .

But as many of my readers may not be acquainted with trigonometry, I shall show
how

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how to find the same sufficiently exact by the scale.

First draw an occult line C D, (fig. 21.) and by help of your scale, make C D, equal 16,6 chains, for so much did C D, in fig. 20, measure, (see field-book) then to lay off the angles at C and D, proceed thus, look in table 4th for the angle at C, viz. $53^{\circ}, 00'$, and against it is 89,2 links, the chord of the angle of 53° , to the radius of 1 chain or 100 links. But this radius being too small to lay off an angle with any degree of exactness; therefore multiply it by 10, (see the last article in part the first, sect. 6th.) then will the radius be 10 chains, and the chord of the angle at C, 8,92 chains: take 10 chains in the compasses from your scale, and with one foot in C describe the arch *ab*: and also with one foot in D, describe the arc *cd*: then

with 8,92 in your compasses and one foot in a make an intersection at b , and thro' b draw an occult line CA . Now by the same method of proceeding we have the chord of $68^{\circ},00'$ the angle at D , equal 11,18 chains, therefore taking this in the compasses, and with one foot in c intersect the arc cd at d , and through d draw Dd , and it will intersect the line CA in A . And if the line AD be measured by the same scale that CD was laid down by, it will be found 15,46 chains; and perhaps as exact as if it had been actually measured on the level surface of the water.

Remark. If a large scale be made use of, the above will be found much more exact than the method of laying down the angles by the common protractor: but if you have a protractor that will lay down angles

gles to a minute, it may be substituted instead of the above, as being rather more expeditious.

Now having found the line AD, we may proceed to find the contents without having any regard to a plan. For the side AB, multiplied by the side or line AD, and the product multiplied by the factor for the including angle BCD, gives 104,547 : and the side BC multiplied by the side CD, multiplied by the factor for the angle BCD, gives 47,216 : and their sum 151,736 is the area of the trapezium ABCD. Again, the side AD, multiplied by the side DE, multiplied by the factor, for the angle ADE, gives 137,559 : and the side EF, multiplied by the side AF, multiplied by the factor for the angle AFE, gives 27,040, and their sum is 164,605, equal the area of the

trapezium D E F A : and the sum of both trapeziums gives 316,368 the whole area within the station lines.

And by finding the area of the several off-sets (as taught in example to fig. 19, in this sect.) we shall have 12,65 for the first line, 1,745 for the second, 10,725 for the third, 29,35 for the fourth, 0,252 for the fifth, and 5,796 for the sixth or last line F A. And the sum of the areas of these off-sets, viz. 60,518 being taken from the whole area within the station lines, leaves the area of the surface of the water equal to 255,85 square chains, or 25 acres, 2 roods, 14 perches.

Now the plot may be had by the 14th Prob. Geom. (as taught in fig. 19.) Or having laid down the triangle A C D (fig. 20.) as directed above for fig. 21. Then with the length of the line A B, 14,05
(see

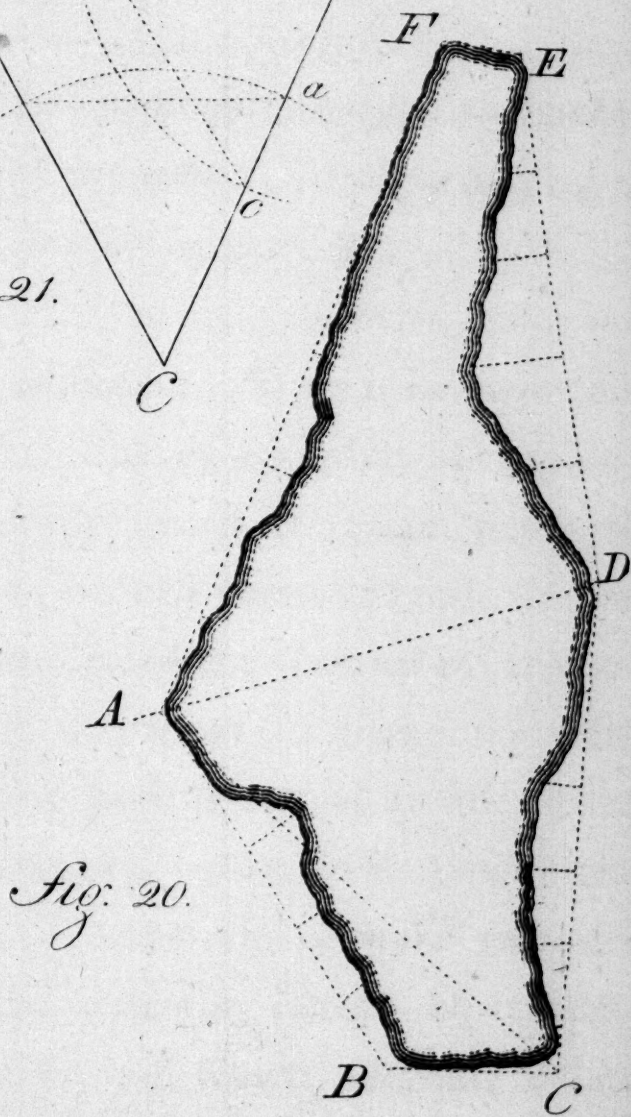
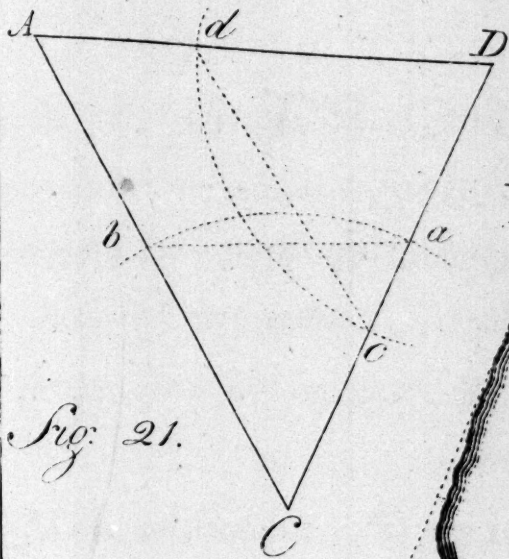
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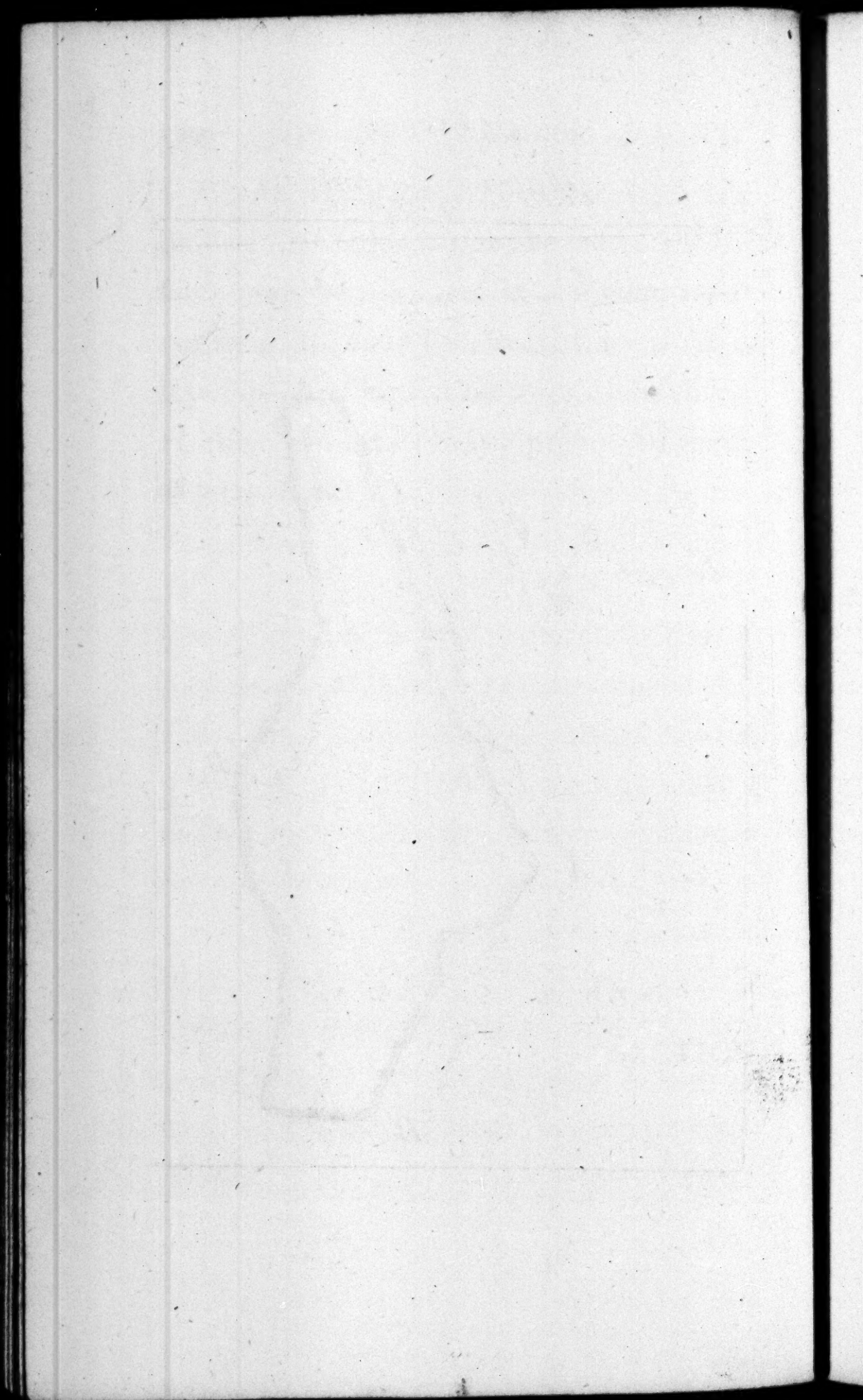
(see the field-book) in your compasses, and one foot in A, describe an arc towards B, also with 5,70 the line B C, and one foot in C, make the interfection at B, draw A B and B C, and it compleats the trapezium A B C D A. And for the other trapezium, first lay off the angle D A F, as taught above in the triangle, (g. 21.) or by the protractor, and draw A F, making it 24,6 chains, then with E F equal 2,3 chains in your compasses, and one foot in F, describe an arc towards E, and with D E, 18, chains in the compasses, and one foot in D make the interfection at E, draw F E and D E, and it compleats the right lined part.

And as for the off-sets, they are to be laid down exactly in the same manner as taught in the last example (fig. 19.) and therefore need not be here repeated again.

Woods which are so filled with trees and bushes, that fights cannot be taken within-side, may be measured in the same manner by going round, endeavouring to do it if possible by four station lines, or by three, as then you may always prove the truth of your work.

SECTION





SECTION VIII.

Shewing how to measure, cast up, and make the plot, of any number of inclosures, such as a farm, parish, lordship, &c.

FOR example, (1st) let it be required to make a survey and plot, of the five inclosures represented in fig. 22. plate 7th.

Having walked over the land, in order to get an idea of the form of it, and to see where it is proper to make the station lines: and finding two of its sides bounded by lanes; and these lanes nearly straight. I conclude to make two of the station lines in the lanes; especially as there is a wood which prevents a station
line

line being made through it; and the other two on the inside the inclosures. And that I can see all the stations from one to another, except the second; which cannot be seen from the first, by reason of a rising ground between them. I therefore set up a mark or white at the gate that goes into the second piece, by the directions given at Prob. 3d, on the use of the chain, from whence I can see the tree at the second station. And having set up whites, or fixed upon marks as trees, &c, for the station lines. I proceed to set up whites at the necessary corners of each inclosure.

Having two assistants, one to lead the chain, the other to follow it. I begin at station the first, which I mark in the field-book thus (□ 1.) And having adjusted my sextant, I take the angle, by bringing the
image

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image of the white or mark at the fourth station, to coincide with the white set-up between the first and second stations; and find it $103^{\circ}, 20'$, which having entered in the field-book, I proceed to measure the first station line; and find at 0,23 chains, I enter the first inclosure, and the off-set on the left hand to the corner is 20 links. Having measured two chains, I take the angles that the first corner of the wood makes, with the (unmeasured part of the) station line; and also the further corner of this piece, and find them $45^{\circ}, 00'$, and $53^{\circ}, 25'$ respectively, see the field-book. At 4,68 chains is an off-set of 40 links, and at 7,20 chains we enter the second field, thro' at gate, an off-set of 36 links on the left to the corner. And here I took the angle that the hedge (between the first and second field) makes with that
part

part of the station line we have to measure ; and find it $92^{\circ}, 00'$, which I enter in the field-book, on the right hand of the column of chains : because the angle was measured on the right hand of the chain line ; and to which I add this remark, viz. *hed. to st.* The meaning of which is, that the image of the hedge, or the white at the corner of the wood, is brought to coincide with the mark or tree at the second station.

And here it may be proper to observe, that if a hedge, a corner, &c. be brought to coincide with the station I have just left ; I make this remark in the field-book, viz. *h. to b. fl.* which means hedge to back station, or angle contained between the hedge and that part of the station line we have just measured. Likewise if the station mark is brought to coincide with an
hedge,

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hedge, a corner, or other object, I write in the field-book *ft. to h. or b. ft. to h. &c.* These abbreviations will be found very useful and easy.

Having measured 9,34 chains, there is an off-set of 20 links to the left ; and here I take the angle that the second corner of the wood makes with the station line, which I enter on the right hand thus, ($\angle 19^{\circ}, 20'. 2d. cor. wd. to ft.$) see the field-book. This being entered in the field-book, I measure on, and at 15,82 chains I find an off-set on the left, of 1,33 chains. And here by way of proof I take the angle that the first corner of the wood makes with the back part of the station line, and find it $29^{\circ}, 25'$, which I enter in the field-book thus $\angle 29^{\circ}, 25'. b. ft. to 1st. cor. wd.$ At 16,34 chains I find we are in a line with the wood hedge, that goes from the
second

second corner straight down to the wood lane, I also take the angle this line or hedge makes with the station line, and find it $69^{\circ}, 00'$. At 19,92 chains I find a perpendicular falls from the left hand corner of this second piece, which measures 4,32 chains. But in measuring this perpendicular, at 1,12 chains from the chain line, I find a perpendicular on the right hand, to the corner of the third piece. Now as this is a perpendicular taken upon a perpendicular, I make a short column on the left hand, in the field-book, as the corner being on that side the chain line (see the field-book) in which I enter 1,12 and against it on the right hand per. cor. a pit, and underneath in the same column I enter 4,32 the whole length of the perpendicular; and also I enter this on the left hand against 19,92 the

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the number of chains at which it fell, with an asterism to shew they both represent the same perpendicular, or off-set at the left hand corner of the second piece. At 20,00 we enter the third piece through an hedge, and here I take the angle that the back part of the station line, makes with the hedge I am going through: which I enter in the field-book thus, $\angle 31^{\circ}, 50'$ b. ft. to hedge. At 21,68 chains is a perpendicular on the left, to the corner of this third piece; but could not be measured conveniently, by reason of a pit. At 22,30 is my second station; I therefore enter the 22,3 chains in the field-book, and against it on one side I write 1ft line; then having drawn a line across the chain column finishes the field-book for the first line.

I now

I now enter ($\square$ 2.) for my second station, and against it I write, in third piece. I then take the angle contain'd between the first and second station lines; and having entered the same, (see the field-book) I proceed to measure the second line. And at 0,00 that is at my second station, there falls a perpendicular from the right hand corner of this piece. And as this line contains nothing more than common off-sets, I shall take no farther notice but refer to the field-book.

Being come to the third station, I enter it in the field-book, and against it I write in lane. I then take the angle contained between the second and third station lines, and having entered it, I proceed to measure the third station line: And at 0,00 is an off-set on the left of 85 links for the breadth of the lane. At 5,40 chains is a
gate

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gate into the third piece; at 5,80 we are against the third corner of the wood, and the off-set to the corner is 14 links, also the off-set to the other side the lane is 22 links. At 20,05 chains is a gate into the wood, and at 20,44 chains we are against the fourth corner of the wood, the off-set to the corner is 11 links. At 29,00 chains we are close to the left hand side of the lane, and at 29,70 is an off-set of 60 links on the right hand to the corner of the fifth piece, and another on the left, to the corners of the lanes of seven links. At 30,20 chains we come to the fourth station.

Here I take the angle, and then measure the station line, and find at 0,80 chains an off-set of 10 links to the corner of the fifth piece, which is a little round, which I remark in the field-book, and

O

at

at 4,00 chains is an off-set of 8 links to the corners of the cross hedge between the fifth and first pieces, which hedge bends in the form of an , which I mark in the field-book, shewing the way it bends, &c. and thus may the form of any intermediate irregular hedge be sketched in the field-book, and plotted sufficiently near the truth, when the lands on each side belong to the same person; otherwise it must be measured and the off-sets taken thereon.]

At 11,00 chains is an off-set on the right hand of 20 links, and on the left to the other side the lane of 45 links. At 13,60 chains we come to the first station again, and the dimensions are finished; see the field-book which now follows.

But before I leave the spot, I add the four angles together, and if they make
360° or

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360°, or very nearly so, I conclude they are rightly taken. And the chain lines will be proved in the plotting.

I also observe the bearing or situation of the land, in respect of north and south, which may be known near enough by the time of day and situation of the sun. Thus at twelve o'clock, I observe (being at station first) the fourth station line makes an angle with an object at a distance, and perpendicularly (or nearly so) under the sun, of 28 degrees, which I enter in the field-book. I also note the names of the owners of the adjacent land, &c.

FIELD-BOOK to Figure 22.

*Dimensions, observations. &c. of part of an
Estate belonging to T. B. of L. Esq; in the
Parish of C. taken April, 1770.*

Off-fets & remarks			Off-fets & remarks
		□ 1ft.	In the lane
	∠	103°,,20'	
to corner	20	0,23	thro' gate & hedge
		2,00	∠ 45°,,00' 1ft. c.
			wd. to ft.
		2,00	∠ 53°,,25' cor.
			cross h. to ft.
	40	4,68	
to corner	36	7,20	thro' gate a hedge
		7,20	∠ 92°,,00' h. to ft.
	20	9,34	∠ 19°,,20 2d. cor.
			wd. to ft.
	133	15,82	∠ 29°,,25' b. ft. to
			1ft cor. wd.
1,12 per. cor. apit.		16,34	in line wood hedge
* 4,32		16,34	∠ 69°,,00' h. to ft.
* 4,32		19,92	
thro' hedge		20,00	∠ 31°,,50' b. ft to
			hedge
Per. cor. a pit.		21,68	
		22,30	1ft line.

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	∠	□ 2.	In 3d piece
		110°,,20'	
		0,00	Per. to cor. this
		8,03	
to corner	40	13,40	
	27	14,21	2d line
	10		
to cor. lanes	∠	□ 3.	In lane
		69°,,40'	
		85 0,00	00
		5,40	a gate into 3d piece
		22 5,80	14 against 3d cor. wd.
		20,05	a gate into wood
		20,44	11 to 4th cor. wood
		00 29,00	
		07 29,70	60 to cor. 5th piece
		30,20	3d line
	∠	□ 4.	In lane.
		76°,,40'	
		50 0,80	10 to cor. ronnd
		4,00	08 against cros h.
		45 11,00	20
		13,60	4th line.

1st $\angle$	103 ^o ,20	
2d $\angle$	110 ^o ,20	
3d $\angle$	69 ^o ,40	
4th $\angle$	76 ^o ,40	
Sum of four angles	360 ^o ,00	a proof of being right
$\angle$ 4th station line	28 ^o ,00	mk.wh. 12 ack.fun.W.

North side against first piece Mr. S's Land.

North and East to $\square$ 3, Mr. B's Land.

The other sides bounded by lanes.

End of the FIELD-BOOK.

N O W the whole content of this is easily obtained from the field-book, without having any regard to a plot, in the very same manner as was the single inclosure, fig. 19, (see Art. 3^o. Sect. 7th.) But in large parcels of land, it is safer to lay down the plot first, in order to discover whether or not any error has been committed

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mitted in the chain lines, that (if any) they may be corrected before we proceed to find the contents: and also for a guide in casting up the off-sets, as observed before in the above-mentioned article.

I shall therefore proceed next to lay down the plot (see Fig. 22.) by a scale of 8 chains in an inch, (as such a scale will bring it within compass, without folding, for folding plates are soon worn out,) but would advise the young surveyor to plot it by scales of 4, and 2 chains in an inch, by way of practice.

To plot the foregoing (or any other like large) dimensions, it will be best to lay down the longest line first. That is, I draw an occult line to represent the third station line, and make it equal 30,20 chains. I then lay off the angle at the third station, viz. $69^{\circ}, 40'$, which if done

O 4

by

by table the 4th of chords, and an half-inch diagonal scale; the angle may be laid down to 5 minutes. And if an inch scale be used the angle may be laid down to $2\frac{1}{2}$ minutes; by observing the rules already given on the use of that table. See the example to Fig. 21. If the angle to be laid down be more than 90 degrees, it will be best to lay it down at twice, especially when great exactness is required.

Note, The half-inch scale is equivalent to a protractor of 10 inches diameter, and the inch scale to 20 inches diameter.

I have been more particular about laying down this angle; because the proof of the chain lines have a dependance on it. As a small error from a small protractor, will be increased into a great one, at the length of the station lines; especially when they are 30, 40, or 50 chains
(or

(or more) long. Having laid down the angle and made the second station line of its proper length, viz. 14,21 chains. I take the length of the fourth station line in the compasses, and with one foot in $\square$ 4th I describe a small arc towards $\square$ 1st, and with the length of the first station line 22,3 chains and one foot in $\square$ 2d, I make an intersection at $\square$ 1st, between which points I draw the station lines.

Now I measure the four angles of the plot, either as above or with the protractor; and find them nearly the same as taken in the field; which is a certain proof of the station lines being truly measured, and entered in the field-book. For a mistake of a chain, &c. in any of the station lines, would vary some of the angles several degrees, and in such case it will

will be necessary to remeasure the lines till the mistake is discovered.

Having laid down the four station lines and proved them, I proceed to lay down the off-sets, and other remarks, by laying the edge of the plotting-scale along the first station line, with σ , on $\square$ 1st, and with the protracting pin, I prick off the several stationary distances as they stand in the field-book. Then turning the scale perpendicular to the station line, and with σ on each stationary distance prick off the several off-sets joining their extremities with a black lead pencil for the bounders on that side. I then lay down the several angles, by laying the center of the protractor to the several stationary distances they were taken at, and the diameter along the station line, with the limb to the right ; I prick off the said angles,

gles, drawing occult lines; the intersection of which, will give the points that those angles were taken to represent.

For if the angle at the second and fourth stationary distances, be thus laid down, their intersections will give the first corner of the wood, and the corners of the 1st and 5th pieces, and by joining these points, viz. the fourth stationary distance, and the points of intersection, with a black lead pencil, gives the representation of the fence between the 1st and 2d pieces, and also part of the wood fence.

Note, It is best to draw the lines representing the bounders or fences, first with a black lead pencil; as then any small mistake committed in drawing those lines, may easily be corrected in drawing them over again with ink.

The

The center of the protractor being laid to the sixth stationary distance, and the angle be pricked off, that the back station made with the first corner of the wood, an occult line being drawn through the angular point will cut the former intersections of the first corner of the wood if the work has been rightly performed, and is a kind of check or proof of the work; and may be often used to advantage in this method of surveying.

The angles at the 5th and 7th stationary distances being laid off, their intersections give the second corner of the wood; and the angles at the 7th and 9th stationary distances, give the first corner of the 3d piece, and in order to find the second corner of it, I lay 0 of the plotting-scale to the 8th stationary distance, with its edge along the off-set line, and at 1,12 chains
make

make a prick, then turning the scale perpendicular to the off-set line, draw from the said prick a short occult line by the edge of the scale to the right hand; then lay the scale to the 10th stationary distance, and perpendicular to the station line, and by the edge of it draw an occult line to the left, and it will intersect the former in the second corner: where the pit prevented measuring to it.

Now having laid down the dimensions of the first station line; I should proceed to lay down those of the second; but as that differs nothing from the side of a single inclosure, I shall refer the young surveyor to the field-book and plot; and proceed to lay down those of the third station line.

By help of the scale (the field-book lying open before me) I prick off the several stationary

tionary distances, as has been already taught; then turning the scale perpendicular to the station line I prick off the offsets, and mark the places for the gates, &c. I then draw a line from the 3d station to the 3d corner of the wood, and from thence to the 2d corner of the wood, continuing it to the 1st corner of the 3d piece, and also from thence to the 2d corner, making there the representation of the pit.

I then continue on the wood fence from the 3d corner to the 4th, and from thence to the corners of the 5th and 1st pieces, I also draw a line from the 1st to the 2d corner of the wood; making them all straight lines, as the fences they represent are such, or very nearly so. I then continue on the fence from the wood to the corner of the 5th piece, by $\square$ 4th. And in the
same

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same manner lay down the off-sets on the 4th station, joining their points, &c. and I also draw the wave line to represent the fence between the 1st and 5th pieces; and by help of the off-sets, the fences on the other sides of the lanes are drawn, and the rough plot finished.

I shall now first find the content of the whole from the foregoing dimensions; and then find the content of each separate piece by the scale; and if the sum of the contents of the latter be equal to the former, I conclude all is right. But if I find a considerable difference, I examine them both over again. I also measure the part within the four station lines with the scale, by taking the diagonal and perpendiculars; and it will discover if any material error is committed in casting up the sides and angles. And if after all this examination

nation and correction, the sum of the separate pieces should differ from the content, found by the first correct method; as they generally will some small matter by reason we cannot be so exact with the scale; I reduce them to that.

By saying (by the Rule of Three) as the sum of the contents by the scale, is to the whole true content, so is the content of each separate piece by the scale, to the true content thereof. Or which is better, take the difference in perches between the true content, and that found by the scale; and say as the true content is to this difference, so is the content of each piece by the scale to a fourth number of perches; which are to be added to or subtracted from such field, according as the sum of the contents by the scale was less, or more than the true content. And thus the true
content

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content of each piece will be had, and their sum exactly agree with the first true content.

And first for the part within the station lines; now 22,3 the 1st station line multiplied by 14,2, the 2d gives 316,66; and this multiplied by ,4688, the factor for the including angle, gives 148,45 the area of the first part. And 30,2 the 3d line multiplied by 13,6 the 4th line gives 410,72 and this multiplied by ,4865 the factor, gives 199,815 the area of the second part, and their sum is 348,265 the area within the station lines.

And the area of the off-sets on the first station line is 24,572: on the second 4,888 and their sum is 29,46 to be added: also 5,673 and 1,867 are the areas on the 3d and 4th lines, and are to be subtracted, being on the outside the bounds of the

P

land

land, as may be seen by the plot (fig. 22), therefore from the sum of the two first 29,46 take the sum of the two last, viz. 7,54 and the remainder 21,92 only, must be added to the area within the station lines, and we have the whole area equal 370,185 square chains, or 37 acres, 0 roods, 03 perches.

Next the area of each separate inclosure is found by measuring them by the same scale they were laid down with, having first (if necessary) reduced them by Prob. 20th. of Geom. to trapeziums, or straightened their sides; as will be more particularly explained by and by. Then measuring the diagonals and perpendiculars, their contents will be found as follows, the first, 6 acres, 2 roods, 03 perches; the second, 6 acres, 1 rood, 30 perches; the third, 7 acres, 3 roods, 17 perches; the

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the fourth, 11 acres, 3 roods, 10 perches;
and the fifth, 4 acres, 1 rood, 09 perches:
their sum 36 acres, 3 roods, 29 perches,
which is less than the first content 37 acres,
0 roods, 03 perches, by 14 perches. Then
say by the rule of three, as 37 acres : 14
perches, :: $6\frac{1}{2}$ (content of 1st piece) : $2\frac{1}{2}$
perches, to be added to the content of the
1st piece, and so of the rest. *Note*, This
proportion is most easily and expeditiously
wrought by the sliding rule : by set-
ting 14 on the slider (which call perches)
to 37 on the rule (which call acres) then
against $6\frac{1}{2}$ acres on the rule, is $2\frac{1}{2}$ perches
on the slider for the answer ; and without
altering the slide, you have the number of
perches answering to any number of acres
by inspection. Thus against 6 acres, 1
rood, 30 perches, the 2d piece or $6\frac{1}{4}$ acres
nearly, on the rule, is $2\frac{1}{2}$ perches as be-

fore on the slider. And against $7\frac{1}{4}$ acres, on the rule (the content of the 3d piece) is 3 perches on the slider. Also against $11\frac{1}{4}$ acres is $4\frac{1}{2}$ perches, for the 4th piece. And against $4\frac{1}{2}$ acres is $1\frac{1}{2}$ perches for the 5th piece. These numbers being added to their respective pieces, their contents will stand thus, 6 acres, 2 roods, $5\frac{1}{2}$ perches for the first; 6 acres, 1 rood, $32\frac{1}{2}$ perches, for the second; 7 acres, 3 roods, 20 perches, for the third; 11 acres, 3 rood, $14\frac{1}{2}$ perches, the fourth; and 4 acres, 1 rood, $10\frac{1}{2}$ perches, for the fifth pieces; and their sum 37 acres, 0 roods, 03 perches, agreeing exactly with the contents found by the first method.

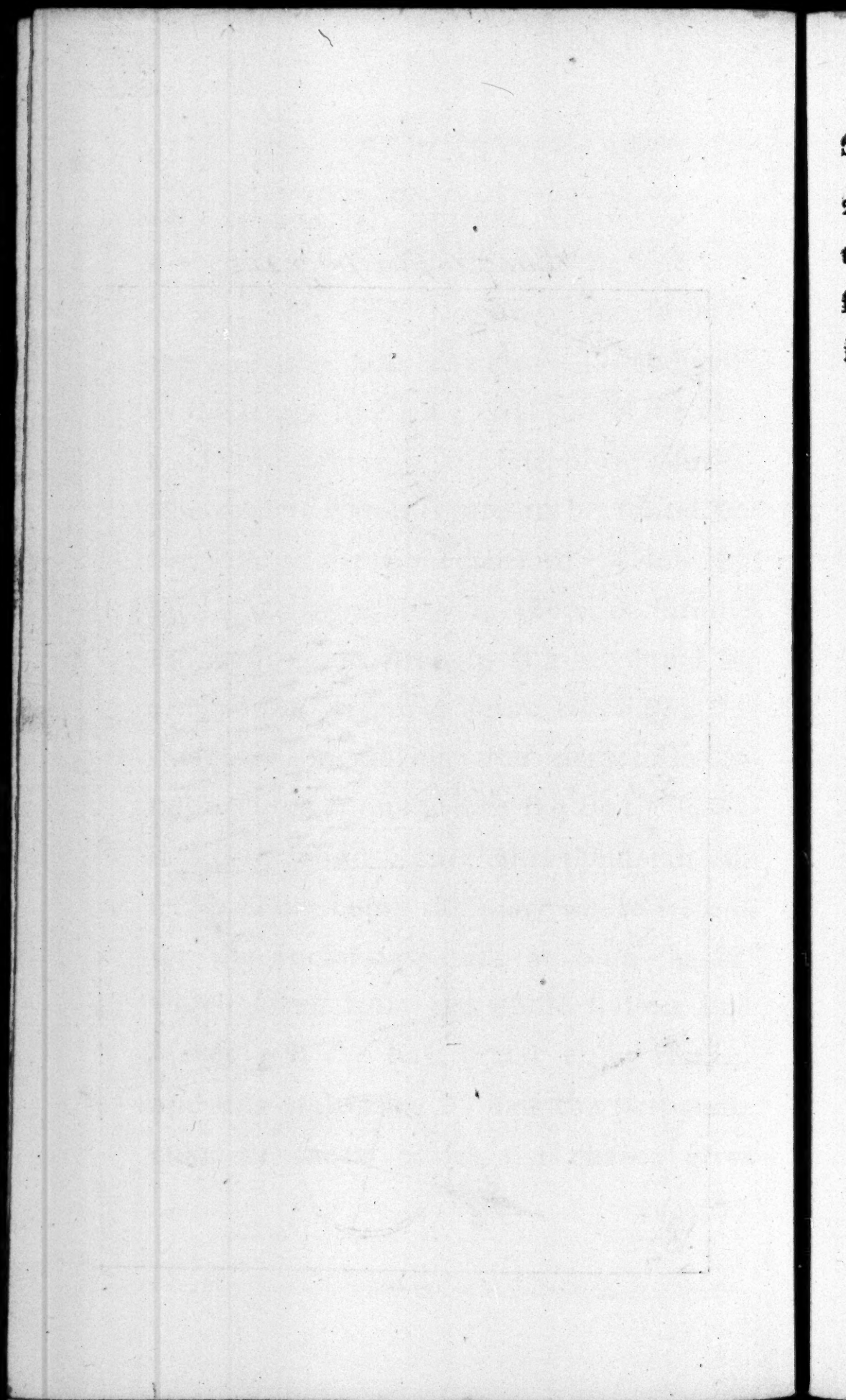
I shall conclude this example with a geometrical method of finding a mean off-set or perpendicular, by which and the
base

base or station line, the content of the off-sets (on the said station line) may be found at one multiplication.

Suppose for example it is required to find a mean off-set to the first station line. In order to make this more plain, I have added fig. 23 from a larger scale, viz. of 4 chains to an inch; then having laid down the off-sets and drawn the bounders as before directed; proceed thus (by Prob. 20, of Geom.) lay a parallel ruler from a (fig. 23) to the next bend but one, viz. to c , then move it up to b , and where the edge of the ruler cuts the line aB , (produced if necessary) make a mark as at h ; lay the ruler to h and d , and move it up to c , and it will cut the line aB in i ; lay the ruler from i to e , and move it up to d , and it will cut aB in k ; lay the ruler to k and f , and move it up to e , and it will

cut aB in l ; lay the ruler to l and g , and move it to f , and it will cut the line aB in m ; lastly draw the line gm ; so shall the irregular side $abcdefg$ be reduced to one right line gm ; and this is the method of straightening the sides of inclosures, when their contents are to be found by the scale as mentioned before. Now the line gm , being drawn, there is formed the right lined triangle Bgm , equal in area to the irregular figure $abcdefgB$; therefore measuring the perpendicular falling from m on the base Bg by the scale, it will be found 2,22 chains the mean off-set. Also g being the place where we enter the inclosures, viz. at 0,23 chains, which taken from the whole station line leaves gB the base equal 22,07 chains, and this multiplied by half the perpendicular or mean off-set 1,11 chains gives

24,4977



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24,4977 square chains for the content of the off-fets ; and very near the same as found before by the other method ; differing only by one perch.

It may not be improper to observe here, that as the base is given from actual measure, and only the perpendicular that is taken by the scale ; the contents may in general be found with sufficient accuracy, and a great deal of trouble saved. And when the utmost exactness is required ; this will be found of great use in proving the truth of the other way.

Remark. I have been informed that in such cases as the above ; some surveyors make a practice of adding the several off-fets together, and dividing their sum by the number of them that was taken ; and the quotient they call the mean off-set ;

by which and the base (or station lines) they find the whole content at one operation as above.

What foundation they may have for this practice, will best appear by working the above example over again by this method ; see the work.

No.	Off.
1st.	0,20
2d	0,40
3	0,36
4	0,20
5	1,33
6	4,32
6)	6,81 (1,13 mean off-set
	6
	8
	6
	21
	18
	3

Here the mean off-set is only two links more than half what it ought to be ; and
confe-

consequently the area or content will be little more than half what it ought to be. Therefore this plainly shews that this method has no place (or at least ought to have none) either in the theory or practice of land surveying.

Indeed it may sometimes happen to give the mean off-set pretty near the truth; and at others to give it double what it ought to be, so there is no certainty; and therefore it ought absolutely to be kicked out of the practice of land measuring; and the geometrical method substituted in its place; which is perfectly true in theory, and very nearly so in practice; and performed with as much ease as the above false method.

Note. If a parallel ruler be not at hand; it may be performed with your plotting scale and a pair of compasses, with the same

same ease, exactness, and expedition ; and which I prefer to the parallel ruler.

Then first lay your scale to the points *a* and *c*, (see fig. 23.) then take in your compasses the nearest distance from *b* to the edge of your scale ; and with this distance let one point of the compasses move gently close to the edge of the scale, while the other traces out a line parallel to it, and cuts the line *a B* in *h* ; then lay the scale to *h* and *d*, and take the distance of *c*, from the scale, and it will cut *a B* in *i*.

In the same manner proceed with the rest.

EXAMPLE II.

If the foregoing example be properly attended to, it will be found to qualify the
young

young surveyor so far, that he can scarce be at a loss in the most difficult case; yet as examples conduce much more to the improvement of youth than precepts; is the reason of my adding this second example. And as it is a near copy of a small tenement or farm, given in WILD'S Practical Surveyor (a book, which I suppose most surveyors are acquainted with) chap. 5th. sect. 1st. fig. 41. It will give a comparative view, and shew the advantage of this my new method above that. In surveying this little farm according to the method in that book; there are no less than twenty-three station lines, and consequently as many angles depending one upon another; consequently on the accuracy of these angles depend the whole work. For if the station lines are measured ever so exact, if the angles are not so likewise, the
plot

plot will not close (as we have before observed) and consequently, neither the plot nor content can be given true.

But according to my method this farm may be measured by four station lines only; for beginning at F, and measuring the station lines F C, C D, D E, and E F, taking the off-sets, &c. as directed in the former example, (see fig. 24, plate 8.) and consequently from these four station lines with their off-sets, and four angles; the whole content and bounding lines of this farm are obtained; without the least fear or doubt of its closing, and with the utmost truth.

In measuring large farms, manors, &c. it is requisite to divide or parcel them out into smaller parts, as may seem most convenient: then measuring these parts one after another till the whole is compleated.

Always

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Always observing to take in as much as conveniently can be at each round joining them together, by making one of the four sides of the first round, the first side of the second round or parcel: and one side of the second round, the first side of the third round, &c. And thus will the whole be connected together. And if proper care be taken in measuring the lines and angles; not only the plot, but the content of the whole will be had with the utmost exactness and truth.

But in order to make this still more plain and easy; I have added the following dimensions of this little farm (see fig. 24.) supposed to be measured in two parts; beginning at A $\square$ 1, and measuring to B $\square$ 2, and so on to C $\square$ 3, D $\square$ 4, and to A $\square$ 1, again; which compleats the first round. (See the field-book for the dimensions

dimensions) Then from A $\square$ 1, to E $\square$ 5, F $\square$ 6, and B $\square$ 2, which compleats the second round.

But as the line E F, lies at a considerable distance from the river, and must have many off-fets taken thereon: it will be better to take only the off-fets between E and H, to this line. Then measuring F I, and G H, taking the off-fets on these lines, to the river; will compleat the dimensions as in the following field-book.

F I E L D.

FIELD-BOOK to Fig. 24.

		□ I. 84° 30'	
	30	2,26	2,18 to lane. + 42 } over lane }
		5,32	1,70 to do. + 40 to c. T. Leas H. Cl.
		8,70	thro' into lane
		9,55	10 to cor. orch.
Gate 30		9,85	260 f. cor. do. 70 45 260
		11,20	20 to cor fold. 60 to } ftab. leng. ft. 100 }
		11,75	20 to gt. 140 to front } house leng. h. 80 }
		12,08	20 to cor. barn 110 length
Pr. c. Cowp: Garrot 42		12,30	20 to 2d cor. do.
		12,90	per. to fur. cor. calves croft.
		13,05	25 to cor. calves croft Stockin.
		14,15 14,25	thro' into stockin

bdth. lane 60 to l. 120	{	14,36	p. 10,20 *
			310 p. cor. cal. c.
			470 thro' in home c.
		gate	70 555
			40 896
			* 1020 to cor. h.cl. hd.
			lit. round to crab close
			$\angle 88^{\circ}, 30'$ bend 1 to ft.
			$\angle 59^{\circ}, 20'$ bend 2 to ft.
		14,36	
		19,50	
		21,85	1 ft line.
	$\angle$	$\square 2.$	
		$94^{\circ}, 40'$	$\angle 51^{\circ}, 30'$ bend 1 to ft.
		0,00	$\angle 32^{\circ}, 40'$ bend 2 to ft.
		3,32	35. $\angle 11^{\circ}, 30'$ bend 3
			to ft.
		6,65	27 to cor. $\frac{\text{stock}}{\text{out wood}}$
		6,85	per. to bend 3.
		13,95	00
		15,95	thro into com. 30 to
			corner
		17,32	100
		18,65	2d line
	$\angle$	$\square 3.$	
		$79^{\circ}, 50'$	
		1,90	60 round
		4,70	00
		10,40	65
		12,55	30

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		14,40	35 to cor. <u>outwood</u> crabclose
		14,40	∠ 86 00 wd. h. to ft.
		15,55	30
		17,95	114
		19,00	thro' into Charlton Fd.
		19,20	68
		20,45	00
		21,92	3d line
	<	□ 4. 101°,,00'	
		300	thro' into crab close
			∠ 49°,,30' cor. Turf.
			L's to ft.
		300	∠ 56°,,30' bend 4 to ft.
95		5,50	
		5,90	per. cor. outwood h.
65		7,30	
		8,60	∠ 75°,,10' b. ft. to b. 4.
00		8,60	thro' into Turf. Lease.
		13,00	∠ 46°,,25' b. ft. to cor.
			this
to cor.	45	13,68	thro' into lane.
		14,70	into cow-pasture
to cor.	32	14,90	
	60	16,52	the 4th line

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Begin again at		□ 1.	for 2d round.
.	<	95°,,30'	
	18	235	
	40	492	
	00	8,30	1st line to river.
		8,70	to furth. side river.
	<	□ 5.	
		91°,,00	
to river	40	1,32	
to river	1,20	2,35	thro' into mag. mead.
		2,35	∠ 53°,,30' bend 5 to ft.
		3,10	1,75
per. to H.	2,90	5,12	∠ 29°,,00' bend 6 to ft.
		8,00	∠ 48°,,50' b. f. to b. 5.
		8,60	per. gate into cow-pasture.
		14,86	*468 to c. this * $\frac{390}{468} \frac{120}{120}$
		14,86	∠ 39°,,30' b. ft. to b. 6.
		16,00	120
		16,10	thro' into Garr. field
to river	00	20,10	
to river	140	21,92	2d line
	<	□ 6.	
		88°,,05	∠ 4°,,10'. 12 o'clock sun to □ 5.
to river	1,30	0,32	and meeting of hedge.
to hedge	32	4,05	

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to corner	50	6,85	
	12	9,55	thro' into lane
	00	10,00	thro' into stockin.
		10,85	3d line, to $\square$ 2.
	<	$\square$ 2. 85°,,20'	ends the 2d round.
	At	$\square$ 6.	an off-set to river at I .
	<	53°,,30'	
to river to do.	70	1,00	84 to river
	45	1,60	80 to river
		2,50	to the river at I .

From G to H an Off-set.

	2,35	1,53 to river
	3,55	1,10
	4,00	thro' into Mag. Mead.
		112 corner
	5,20	1,35
	7,40	20
	9,53	95
	11,65	00 Bridge
	14,30	15
	15,20	00 to H .

1ft. < 84°,,30'
2. < 94°,,40'
3. < 79°,,50'
4. < 101°,,00'
360,00

1ft. < 95°,,30'
2. < 61°,,00'
3. < 88°,,05'
4. < 85°,,20'
359°,,55'

The adjoining lands are, north side JOHN CLEMSON's on the east side the road, and WILLIAM GREEN's on the west side the road. East side is *Charlton Common*; south side is *Charlton Field*; west side is HENRY BRANSTON's land.

And thus ends the FIELD-BOOK.

Now to make the plot.

Having a large sheet of cartridge paper (which is best for drawing the rough maps on, and if one sheet is not large enough, paste more to it with paste) draw the line A B making it 21,85 chains, from the half-inch scale; which I advise the young surveyor always to use in laying down the rough plan. Then lay off the angle at B, by help of the scale and table of chords, making it $94^{\circ}, 40'$, draw B C
of

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of its just length ; and with C D and A D find the point D (by Prob. 14. Geom.) draw C D and D A, and the station lines to the first round are done.

Lay off the angle B A E $95^{\circ}, 30'$ which the line A E makes with A B, and draw A E of its just length ; then with E F and B F find the point F, and draw E F and B F, which compleats the station lines to the second round. And in the same manner may any number of rounds be laid down.

And as for the off-fets and other remarks they are to be laid down as they occur on each station line ; just in the same manner as in the foregoing example ; and therefore need not be here repeated. Especially as the field-book compared with fig. 24, will better direct the surveyor, than a multitude of verbal explanations

can. Therefore shall leave that for the exercise and practice of the young surveyor.]

f The content is also found in the same manner as directed for the former examples, only observing here, that the line A B is to be used in casting up both rounds. Thus A B multiplied by B C, multiplied by the factor for the angle B, gives 203,0177; and C D multiplied by D A, multiplied by the factor for the angle D, gives 177,7278 their sum 380,7455 is the content in square chains of the first round. Also A B multiplied by B F, multiplied by the factor for the angle A B F, gives 118,1332; and A E multiplied by E F, multiplied by the factor for angle E, gives 95,3329; their sum 213,4661 is the content of the second round,

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round, &c. And the content of the whole will be found nearly as follows, viz.

Trapez. A B C D	380,7455 chs.	
Do. A B F E	213,4661	
Triang. E G H	29,145	Off-sets subtract.
Off-sets G H	9,12	B C & C D 15,34
FI	2,5	the road 8,92
D A & A E	12,92	<u>24,26</u>
Sum Additive	647,8996	<u>24,26</u>
Sum Subtractive	24,26	
Whole Content	623,6366	

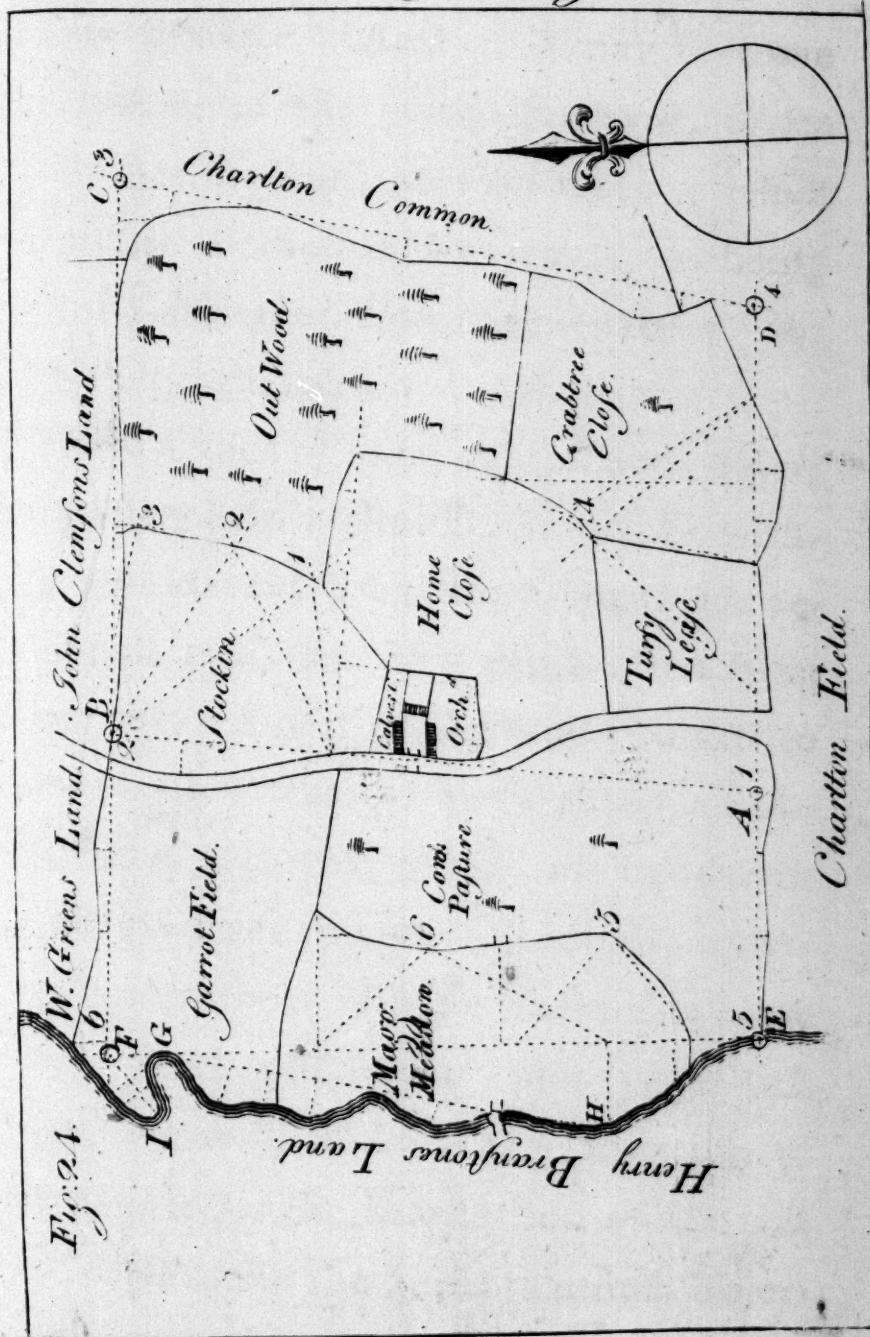
or 62 acres, 1 rood, 18 perches. This differs a little from the content in WILD's book ; which may easily happen, as these dimensions were only copied from his plate by help of a scale, and not taken in the fields.

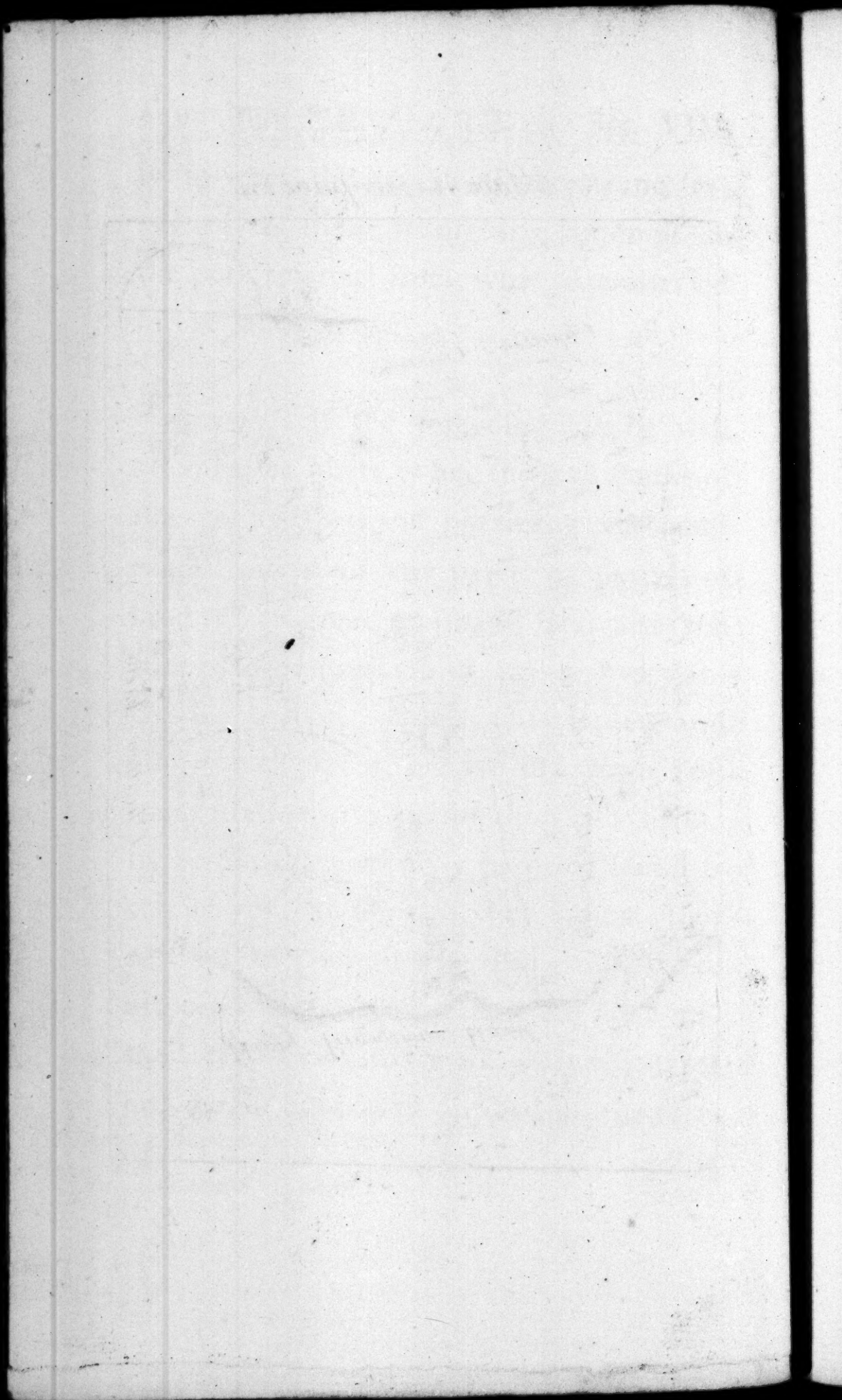
I have omitted inserting the contents of each separate inclosure, and their comparison with the sum of the whole ; purpose-

ly to whet the genius of the young surveyor ; as there is nothing different in the performance from what was directed in the last example.

Remark. If the foregoing rule be well attended to, there is not the least doubt of the young surveyor obtaining the most exact content of any parcel or parcels of land either great or small ; and also the true plan or situation of any or every inclosure therein. But as some writers and also some practical surveyors have laid great stress on the use of the Plain Table, by reason the work may be proved as it is carried on ; and which they say no other method will admit of. But in this they are certainly mistaken ; for the same method of proving the work as it is carried on, might have been applied in the foregoing

Plate 3 to face page 232.





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going examples, or any other that may occur in practice. And as it may be some satisfaction to the young surveyor, to see daily that his work goes on right. I shall insert the method in few words, as being nothing different from some parts of the foregoing examples.

Make choice of some conspicuous object as the steeple of a church, (if such be within your survey) a chimney, or large remarkable tree, &c. that can be seen from all or greatest part of the land you are to survey; then take the angle that such object makes with two of the most convenient station lines in your first round, entering them in the field-book where they were taken; and when you plot your first round, you will find the true situation of such object by the intersection of the angles. Also take the angle that such object makes

makes with one of your station lines in the second, third, &c. rounds ; these angles when plotted (one day after another) will all intersect each other in the place of the object, if your work be right. Otherwise some mistake hath been committed, and must be corrected before you proceed.

N. B. If your survey be large, and one object cannot be seen from the greatest part of it ; you may use two, three, &c. but the fewer the better.

SECTION

SECTION IX.

Giving directions for reducing hypotenusal to horizontal lines.

IN the foregoing examples the land was nearly horizontal ; but in some parts of the kingdom the surveyor will often meet with hills and vales in the course of one or more long lines ; as I have experienced in *Worcestershire, Herefordshire, Derbyshire, and Wales.* And when we meet with such hills or vales in a survey, we are often obliged to measure over the hills, and through the vales ; and therefore such
lines

lines will often be considerably longer than the true base or horizontal lines.

Therefore in plotting such lines, as we cannot make a convex or concave superficies upon a piece of plain paper; we must reduce the hypothenuſal to horizontal lines. For it is only these horizontal or base lines that ought to be laid down in the plot, that every field therein may lie in its true situation.

And in order to obtain these horizontal lines, we must take the angles of the hill's elevation. Suppose fig. 25. plate 9, to represent a hill in the course of one of your station lines; and being arrived at the foot of it at A, 20 chains from your station; let a mark be set on the top at B, just of the same height with your eye; then take the angle of altitude B A D, $20^{\circ}, 00'$, and enter it in your field-book
against

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against 20 chains and note what it is for; measure up the hill to B, and suppose it at 25,34 chains from your station, enter it in your field-book; continue your measure down the other side to C, 29,56 chains from your station, and there take the angle of altitude B C E $18^{\circ},00'$, which enter against 29,56 chains, and continue your measure to the next station.

Now to plot these observations, draw an occult line as A D, and suppose it part of your station line, and the point A, at 20 chains, lay the center of your protractor to A, and diameter to the line A D; prick off the angle D A B $20^{\circ},00'$, and through this point draw an occult line A B; then from 25,34 your chain line at B take 20, your chain line at A, and there remains 5,34 chains, which set from A to B, and from B let fall the perpendicular

cular B D upon A D, then lay the center of the protractor to B, and diameter on the line B D, and prick off the angle D B C $72^{\circ}.00'$, (being the compliment of B C E, $18^{\circ}.00'$ to 90°) and through this point draw the occult line B C, making it 4,22 chains, being the difference between the chain line at B and C; and from C let fall the perpendicular C E, upon B D, so shall A D added to E C or D F be the true horizontal line.

But the line A C is the base line of the hill, and is what ought to be used both to lay down the plot, and to find the content of the whole round; unless the hill extends quite across the opposite station line, and then the whole lines as measured over the hill ought to be used in finding the content, and such base lines as A C for the plot.

And

And here it may be proper to observe, that if the land on both sides the hill, had been level, or of the same perpendicular distance from B, then the point C and E would have coincided with F and D. And then A C and A F would be equal.

If the horizontal lines are measured by the same scale as used to lay down the plot, A D will be 5, and E C or D F 4 chains.

It will also be proper to shade over that part of your plot, where the lines are so reduced, with the representation of the hill, lest another person should measure such part of the plot by the same scale, and find the contents to differ.

If the area of a pointed hill, bounded by an irregular polygon be wanted ; the method of surveying by going round it, will give the area too small ; giving only the

the area of the hill's base. Therefore in such cases, it is best to measure it from one or more stations on the top of the hill, thus: Let A B C D E F G (fig. 26, plate 9.) be the irregular bounders of an hill, and suppose S your station on the top; then having set up marks at A, B, C, D, &c. take the angles A S B, B S C, C S D, &c. and measure the lines S A, S B, S C, &c. Now these lines and angles constitute seven triangles; in which two sides and the including angle of each are known, and consequently the area of each is easily had from the foregoing work, and the sum of these areas will give the area or superficies of the hill.

If such hill is to be plotted with the adjoining lands, then the horizontal or base lines must be found; either by measuring round it; or by taking the hills altitude
at

at A, B, C, D, &c. and such lines made use of in laying down the plot. But if only the plot of the hill is wanted, it may be laid down from the measured lines near enough. In either cases proceed thus :

Choose a convenient point on your paper to represent your station on the hill, as S, and draw S A, and from your scale make it its proper length ; then with the center of your protractor on S, and diameter to S A, prick off the angle A S B, and make S B its just length, join A B, for the bounding side of the first triangle. Lay the center of your protractor to S and diameter to S B, and prick off the angle B S C, and make S C its just length, join B C for the bounding line of the second triangle. And proceed in the same manner with the rest till you get round.

R

N. B. This

N. B. This method of surveying from a station near the middle, may sometimes be of use in single inclosures; when some obstructions such as wood, water, &c. may prevent measuring round. And if all the corners or bends cannot be seen from one station, two or more may be used; observing to measure the distance between each station, and the angle such line makes with one of the lines measured from your foregoing station to one of the corners.

Remark. Authors and surveyors differ in opinion, relating to the measuring of hills; some arguing, that no more corn, grafs, &c. can grow on a hill, than on the base or flat, if the hill was taken away, for say they, as many pales set perpendicularly, as will fence the base or horizontal line, will also fence over the hill; granted.

But

But the same length of railing that fences the horizontal line, will not fence over the hill; nor does corn or grass grow perpendicular. Suppose the side A B, of a hill whose slope line or hypotenuse be 5,34 chains, (as in fig. 25.) and a part of this be cultivated, suppose 2 chains broad; then the area will be 106800 square links, and suppose it to be sown with onions or carrots, and that they grow at one link (or 8 inches very nearly) distance from each other; then there will be 106800 onions or carrots on such a spot. But the horizontal line A D of such hill is but 5 chains, and consequently the area for 2 chains breadth will be but 100000 square links, or just one acre; and will produce only 100000 onions or carrots at the same distance as above. Now the difference of the quantity of onions or carrots on the

hill, and on the flat is 6800, and suppose them worth one penny a dozen, it amounts to 566 pence, or 2l. 7s. 2d. which sum is sufficient to pay the rent of more than double the whole quantity of land; and plainly demonstrates, that surveyors ought always to give in the area of the convex surface of hills.

SECTION

S E C T I O N X.

On measuring common fields.

W H E N ploughed lands in common fields are measured by the chain, 'tis usual to measure the length down the ridge of the land, and to take the breadth at the top of the land, about the middle, and at the bottom; and adding these three numbers together, to take the third part of the sum for the mean breadth: but it is not adviseable to take the breadth very near the land's ends, because the turning of the plough generally makes it considerably narrower or wider; and if in measuring down the land you find the breadth is not nearly equal; it is best to

R 3

measure

measure cross the land oftener, as at every three or four chains length, and adding the several breadths together, divide their sum by the number of breadths, for the equated breadth.

'The several persons lands in common arable fields, may be accounted as so many particular enclosures, and measured after the same manner, by setting up marks at the extremities of each person's land, and measuring the angles from your station lines, taking the off-sets to each man's particular lands; and against such off-set write the name of the owner or tenant; and when you plot such land, you may by help of your field-book express each particular man's land in your draught, with its buttings and boundings, &c. An example of this seems needless, as it is nothing different from some of the foregoing work.

SECTION.

S E C T I O N X I.

Shewing how to lay out any given quantity of land, in any proposed figure.

AS the quantity of land is generally given in acres, roods, and perches; the first thing to be done is to reduce it to square chains.

P R O B L E M I.

To reduce any number of acres, roods, and perches, to square chains.

R U L E I.

To the right hand of the given acres place a cypher; find the roods and

R 4 perches

perches in the 5th table, or the nearest to them less, in the columns of roods and perches; and take out the number of square chains and decimals, standing against them, in the first column, and also the number against the remaining perches, (if any) which numbers added together, gives the square chains and decimals required.

EXAMPLE 1. Let it be required to reduce 6 acres, 2 roods, 14 perches, to square chains :

60 = the acres with a cypher prefixed.

5,000 the number for 2 roods.

,875 ditto for 14 perches.

65,875 The square chains required.

EXAMPLE

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EXAMPLE II. Let it be required to reduce 2 acres, 3 roods, 26 perches, to square chains :

20 acres with a cypher.
9,000 for 3 roods, 24 perch. nearest less.
,125 for the 2 perches remaining.
29,125 the square chains.

EXAMPLE III. Let 20 acres, 0 rood, 20 perches, be reduced to square chains :

200 acres with a cypher.
1,00 for 16 perches, nearest less.
,25 for 4 perches remaining.
201,25 square chains.

R U L E II.

Write down the acres, roods, and perches, each one under the other, beginning with the perches, under them write the

the roods, and under them the acres ; divide the perches by 4, writing the quotient to the right hand of the roods, which also divide by 4, writing the quotient to the to the right of the acres ; and they will be reduced to square chains.

EXAMPLE I. Reduce 2 acres, 3 roods, 26 perches, to square chains :

$$\begin{array}{r|l} 4 & 26 \\ 4 & 36,5 \\ & 29,125 \end{array} \text{ Answer.}$$

EXAMPLE II. Reduce 26 acres, 0 roods, 12,46 perches, to square chains :

$$\begin{array}{r|l} 4 & 12,46 \\ 4 & 03,115 \\ & 260,77875 \end{array} \text{ Answer.}$$

EXAM-

SEC. XI. LAYING OUT LAND. 251

EXAMPLE III. Reduce 7 acres, 0 roods, 01 perches, to square chains :

$$\begin{array}{r|l} 4 & 01 \\ 4 & 00,25 \\ & 70,0625 \end{array} \text{ Answer.}$$

N. B. If you would have square links multiply the square chains by 10000 gives square links.

P R O B L E M II.

Upon a given base to lay out a triangle of any number of acres, &c.

R U L E.

Divide the area in square chains, by half the base ; and the quotient will be the perpendicular.

EXAMPLE

EXAMPLE. Lay out 2 acres, 2 roods, 34,56 perches, in a triangle whose base is 9,36 chains :

$$\begin{array}{r}
 \text{square chains} \quad 4 \mid 34,56 \\
 \quad \quad \quad 4 \mid 28,64 \\
 \quad \quad \quad \hline
 \quad \quad \quad 27,16 \\
 \quad \quad \quad \hline
 \quad \quad \quad 4,68
 \end{array} = 5,80 \text{ chains the perpen.}$$

Then upon any part of the given base A B, (fig. 27, plate 9.) erect a perpendicular C D, by help of your sextant (or some other instrument), measuring thereon 5,80 chains to D; and stake out the lines D A, and D B, and it is done.

PROBLEM

P R O B L E M III.

Upon a given base to make a rectangle of any content proposed.

R U L E.

Divide the content by the base, from each end raise a perpendicular equal to the quotient; and join their tops with a right line, which must be equal to the base.

EXAMPLE. Let it be required to lay out 1 acre, 1 rood, 21 perches, or 13,8125 square chains; in a rectangle whose base AB (fig. 28.) is 5,40 chains.

$$\text{now } \frac{13,8125}{5,40} = 2,56 \text{ chains the perpendicular } B C \text{ equal } A D.$$

P R O B L E M

P R O B L E M IV.

To lay out any quantity of land proposed in a square.

R U L E.

Extract the square root of the proposed area, and it will be the side of the square; measure a right line equal to this root; raise a perpendicular on each end equal to it, and join the tops of these: Make all these lines, and you have done.

EXAMPLE. Let it be required to lay out 5 acres, 2 roods, 00 perches, or 55 square chains in a square.

Now the square root of 55 c. is 7,416 chains for the length of the side of the square, = A B, = B C, = C D, = D A.
Fig. 29.

P R O B L E M

P R O B L E M V.

Upon a given base, to lay out a rhombus of any content less than the square of it.

R U L E.

Divide the content by the base, and the quotient will be the perpendicular. Then from the square of the base, subtract the square of the perpendicular, and find the square root of the remainder. From one end of the base measure a line thereon equal to this root; and at this point erect and measure the perpendicular; at the end of which also erect another perpendicular, and measure a line thereon equal to the base; join the ends of this line, and the ends of the base, and it's done.

EXAMPLE.

EXAMPLE. Let 3 acres, 3 roods, 34 perches, be laid out in a rhombus, whose base A B (fig. 30.) is 7,20 chains. Now 39,625 square chains, divided by 7,2, quotes 5,5 chains for the perpendicular; and the square of 7,2 = 51,84, and of 5,5 = 30,25 their difference = 21,59 whose root is 4,646 chains. Measure from A to to E 4,646 chains, and erect the perpendicular E D, making it 5,5 chains, erect the perpendicular D C, measuring thereon 7,2 chains, join C B and D A, which compleats it.

P R O B L E M VI.

To lay out a trapezium of any proposed content, having one side, or a base given.

R U L E.

Divide the content into two parts, either equal or unequal; then lay out one part
in

in a triangle by Prob. 2. Let the perpendicular of this, be a base to lay out the other part in a triangle by the same, (Prob. 2) and this said perpendicular will be the diagonal of the trapezium.

N. B. In the first part the perpendicular must be erected at the end of the given side, but in the second, its situation may be taken at pleasure ; unless two of the sides of the trapezium has fixed bounders.

EXAMPLE. Let 9 acres be laid out in a trapezium, having one side or base A B, (fig. 31.) 8 chains. Now suppose the two parts to 5, and 4 acres ; then $\frac{50}{4}$ chains, = 12,5 chains, the perpendicular B D, which is also the diagonal. And $\frac{40}{6,25} = 6,40$ the perpendicular E C ; then joining
S
A D,

A D, D C, C B, and the trapezium is completed.

P R O B L E M VII.

To lay out any given quantity of land in a regular polygon.

R U L E.

Find the area of a polygon, of the same name with that required, the side of which is, 1. (in the following table) Divide the proposed area by this area, and extract the square root of the quotient; this root is the side of the polygon required; which may be traced out by Prob. 16, of Geometry.

Or thus having found the side, multiply it by the polygon's number (in the table)

for

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for the radius of the circumscribing circle; and it gives the radius circumscribing the required polygon. With this radius draw the circumference of a circle, and as many chord lines therein, as the number of the polygon requires, each equal the length of the side.

A TABLE of regular polygons, with the area, and radius of the circumscribing circle, and the side of the polygon is 1.

No. fides	Names.	areas.	radius's
3	Triangle	0,433	0,577
4	Square	1,	0,707
5	Pentagon	1,72	0,851
6	Hexagon	2,598	1,
7	Heptagon	3,634	1,152
8	Octagon	4,828	1,306
9	Nonagon	6,182	1,462
10	Decagon	7,694	1,619
11	Undecagon	9,365	1,775
12	Duodecagon	11,196	1,932

EXAMPLE. Let 2 acres, or 20 square chains be laid out in the figure of a regular octagon. Now 20 chains divided by 4,828 the tabular number for the octagon, gives 4,1425 for the square of the side, whose root 2,035 is the side required; and this multiplied by 1,306 the radius in the table, gives 2,658 nearly for the radius of the circumscribing circle; with this radius describe the circle, (fig. 32.) in which draw the chord lines A B, B C, C D, D E, E F, F G, G H, and H A, each equal 2,035 the polygon's side.

P R O B L E M VIII.

To lay out any quantity of land in a circle.

R U L E.

Divide the area by ,7854 and extract the square root of the quotient for the diameter,

SEC. XI. LAYING OUT LAND. 261

meter, the half of which is the radius ;
by which describe the circle.

EXAMPLE. Let 1 acre or 10 square
chains be laid out. Now $\frac{10}{.7854} = 12,7323$
and the square root of this is 3,568 the
diameter, and the half is 1,784 chains the
radius A C, (fig. 33.)

P R O B L E M IX.

*Having one diameter given (either the long or
short one) to make an ellipsis of any proposed
content, not exceeding three fourths of the
square of the diameter.*

R U L E.

Divide the content by ,7854 and the
quotient by the given diameter, the last

P 3

quo-

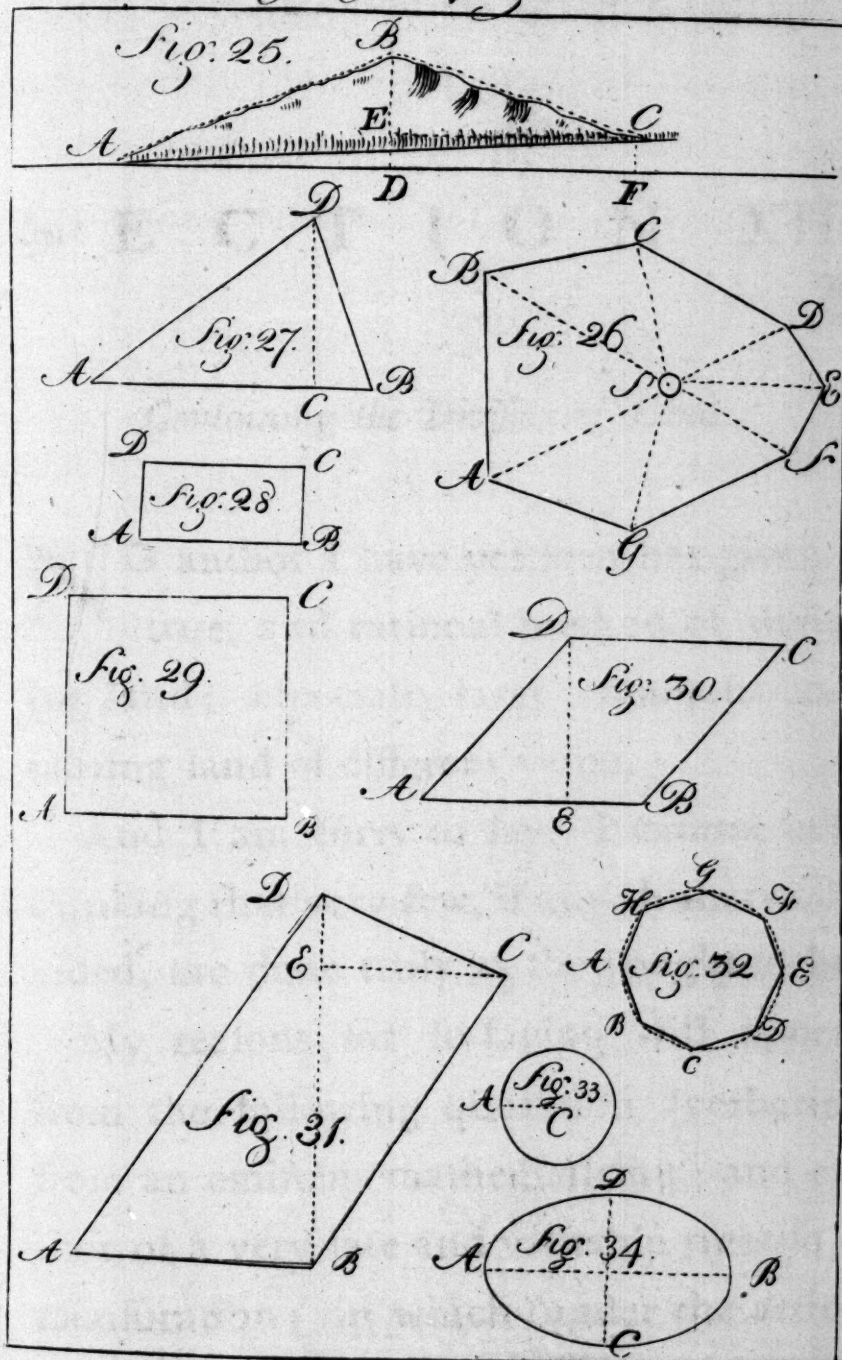
quotient is the other diameter ; then lay out the ellipsis by Prob. 18th of Geometry.

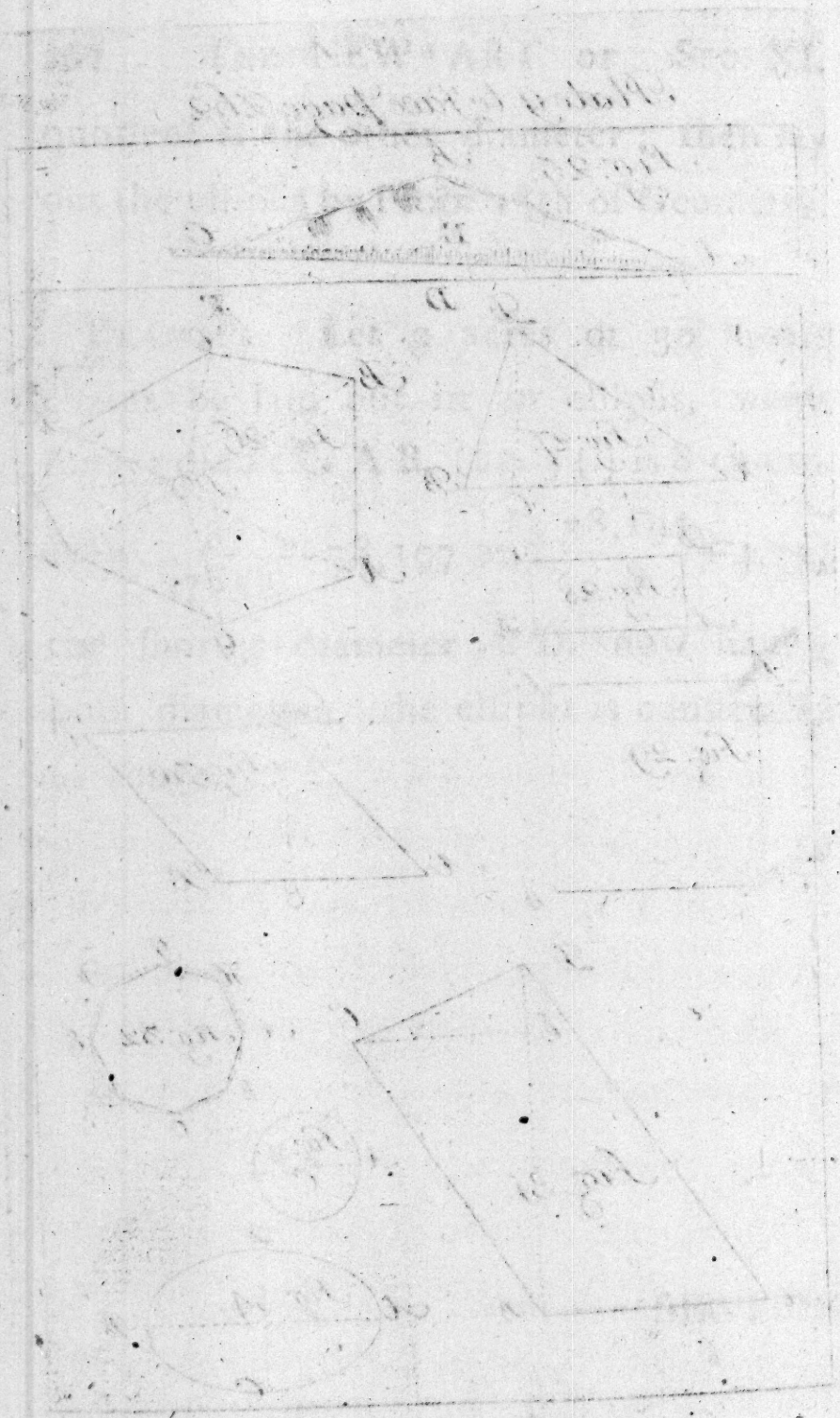
EXAMPLE. Let 3 acres or 30 square chains be laid out in an ellipsis, whose longer diameter A B, (fig. 34.) is 8 chains.

$$\text{Now } \frac{30}{.7854} = 38,197 \text{ and } \frac{38,197}{8} = 4,774$$

the shorter diameter C D, now having both diameters, the ellipsis is constructed as above.

Plate 9 to face page 262.





S E C T I O N XII.

Containing the Division of Land.

NO author I have yet seen has given a true, and rational method of dividing land; especially large commons containing land of different value.

And I am sorry to say, I cannot help thinking that very few, if any, hitherto divided, are done truly as they ought to be.

My reasons for so saying will appear from the following quotation (verbatim) from an eminent mathematician, and author of a very late and valuable treatise of mensuration; in which (under the article

Surveying, chap. 5.) he says, “ In the division of commons, after the whole is surveyed and cast up, and the proper quantities to be allowed for roads, &c. deducted, divide the neat quantity remaining among the several proprietors, by the following problem, in proportion to the real value of their estates, and you will thereby obtain their proportional quantities of the land. But as this division supposes the land, which is to be divided, to be all of an equal goodness, you must observe, that if the part in which any one's share is to be marked off, be better or worse than the general mean quality of the land; then you must diminish or augment the quantity of his share in the same proportion as near as you can judge.

PROBLEM

P R O B L E M I.

“It is required to divide any given quantity of ground into any given number of parts, and in proportion as any given numbers.”

R U L E.

“Divide the given piece after the rule of fellowship, by dividing the whole content by the sum of the numbers expressing the proportions of the several shares, and multiplying the quotient severally by the said proportional numbers for the respective shares required.”

EXAMPLE. It is required to divide 300 acres of land among A, B, C, and D, whose claims upon it are respectively in
 propor-

proportion as the numbers 1, 3, 6, 10, or whose estates may be supposed 100*l.* 300*l.* 600*l.* and 1000*l.* per annum.

The sum of these proportional numbers is 20, by which dividing 300 acres, the quotient is 15 acres, which being multiplied by each of the numbers 1, 3, 6, 10, we obtain for the several shares as follows:

	<i>a.</i>	<i>r.</i>	<i>p.</i>
A's share =	15	: 0	: 00
B's share =	45	: 0	: 00
C's share =	90	: 0	: 00
D's share =	150	: 0	: 00

Sum = 300 : 0 : 00 the proof.

But this is upon supposition that the land is all of an equal value.

Now let us suppose the land to be laid off for each person's share, is of the following different values per acre, viz. A's = 5*s.* B's = 8*s.* C's = 12*s.* D's = 15*s.* an acre; whose sum 40*s.* divided by 4, their number,

SEC. XII. DIVIDING LAND. 267

ber, quotes 10 s. for the mean value per acre. And according to the author just quoted, we must augment or diminish each share as follows ;

$$\text{as } \left\{ \begin{array}{lcl} \begin{array}{cc} s. & s. \\ 5 : 10 :: & 8 : 10 \end{array} & :: & \begin{array}{cc} a. & a. \\ 15 : 30 = \text{A's share.} \\ 45 : 56,3 \text{ B's share.} \\ 90 : 75 \text{ C's share.} \\ 150 : 100 \text{ D's share.} \end{array} \end{array} \right.$$

$$\text{sum of shares} = 261,3 \text{ acres}$$

which is less than 300, by more than 38 acres, and shews the rule to be absolutely false ; for when each person's share is laid out as above, there remains 38 acres unapplied ; I suppose for the use of the surveyor.

But suppose we change the value of each person's land, and call A's 15 s. B's

12 s.

12s. C's 8s. and D's 5s. then we shall have

	s.	s.	a.	a.	
as	15	: 10 ::	15	: 10	A's share.
	12	: 10 ::	45	: 37,5	B's share.
	8	: 10 ::	90	: 112,5	C's share.
	5	: 10 ::	150	: 300	D's share.

sum of the shares 460 acres.

Pray how must the surveyor manage it here? as he must make 460 acres of 300, for here D's share only takes the whole 300, where must the poor surveyor find the 160 acres for the other three shares. But enough of this; see it truly and methodically performed in the next problem.

N. B. The above rule is the only one I have ever seen taught or laid down by any author; and had there been a more correct one known, I make no doubt but the above quoted author would have added it
along

along with the many other valuable improvements to be met with in his before-mentioned treatise.

P R O B L E M II.

It is required to divide any given quantity of land, among any given number of persons, in proportion to their several estates, and the value of the land that falls to each person's share.

R U L E.

Divide the yearly value of each person's estate by the value per acre, of the land that is allotted for his share, and take the sum of the quotients ; by which divide the whole given quantity of land, and this quotient will be a common multiplier ;
by

by which multiply each particular quotient, and the product will be each particular share of the land.

Or say as the sum of all the quotients, is to the whole quantity of land; so is each particular quotient, to its proportional share of the land.

EXAMPLE. Let 300 acres of land be divided among A, B, C, D, whose estates are 100*l.* 300*l.* 600*l.* and 1000*l.* respectively per annum; and the value of the land allotted to each is 5, 8, 12, and 15, shillings an acre, as in the example to the first problem.

$$\text{Then } \frac{100}{5} = 20, \quad \frac{300}{8} = 37,5 \quad \frac{600}{12} = 50,$$

$$\text{and } \frac{1000}{15} = 66,666, \text{ and the sum of these}$$

quotients

quotients is 174,166 $\therefore \frac{300}{174,166} = 1,72248$

the common multiplier :

		<i>acres.</i>	
then	{	$1,72248 \times 20$	$= 34,45$ A's share.
		$1,72248 \times 37,5$	$= 64,593$ B's share.
		$1,72248 \times 50$	$= 86,124$ C's share.
		$1,72248 \times 66,666$	$= 114,832$ D's share.

sum of the shares $= 299,999$ acres, or 300 ;
very nearly, and is a proof of the whole.

Let us now change the values of the land as in the first problem, and see what each will have for his share. Suppose A's 15, B's 12, C's 8, and D's 5, shillings an

acre ; then $\frac{100}{15} = 6,666$ $\frac{300}{12} = 25$, $\frac{600}{8} = 75$,

$\frac{1000}{5} = 200$, and the sum of these quotients

is 306,666, therefore $\frac{300}{306,666} = 0,97826$

the common multiplier :

then

$$\begin{array}{rcl}
 & & \text{acres.} \\
 \text{then } \left\{ \begin{array}{l} 0,97826 \times 6,666 = 6,5217 \text{ A's share.} \\ 0,97826 \times 25, = 24,4565 \text{ B's share.} \\ 0,97826 \times 75, = 73,3695 \text{ C's share.} \\ 0,97826 \times 200, = 195,6520 \text{ D's share.} \end{array} \right.
 \end{array}$$

the sum of the shares = 299,9997 acres, which proves the whole to be right.

N. B. It may be proper to observe here, that as the value of C's land is less than the mean value, his share ought (according to the before-quoted author) to be augmented above the mean quantity, viz. 90 acres, as in problem the first; but here you see, in reality it is diminished, being little more than 73 acres; and further shews the absurdity of the first rule in dividing land of different value.

EXAMPLE II. Let 500 acres be divided among six person's, whose estates are as follow, viz. A's 40*l*. B's 20*l*. C's 10*l*.
D's 100*l*.

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D's 100*l.* E's 400*l.* and F's 1000*l.* per annum, and the value of the land most convenient for each is A's, B's, and C's, each 7*s.* D's 10*s.* E's 15*s.* and F's 12*s.* an acre; now each estate divided by the value of his share of the land, will stand thus :

	s.	l.		
A	7)	40	(	5,71428
B	7)	20	(	2,85714
C	7)	10	(	1,42857
D	10)	100	(	10,
E	15)	400	(	26,66666
F	12)	1000	(	83,33333

quotients.

sum of the quotients = 129,99998 or 130.

Now $\frac{500}{130} = 3,846153$ the common multiplier; by which multiplying each quotient, we shall have for each share as follows, viz.

T

acres.

	<i>acres.</i>
A =	21,9778
B =	10,9918
C =	5,4941
D =	38,4615
E =	102,5641
F =	320,5127

their sum is = 500,0020 & proves the whole.

N. B. If any single share should contain land of several different values, then use the mean to divide his estate by.

Also if there be different quantities as well as values, find what each quantity is worth at its value, and add their sums together ; then say as the sum of the quantities, is to this sum ; so is one acre, to its mean value, to be made use of.

Now having found each person's share, they may be laid out in any form required, by the directions and problems given in the last section. But when several shares contain

contain land of the same value, it is best to lay out their sum in the most convenient form, and then to subdivide it, by some of the following problems.

P R O B L E M III.

To divide a triangle in any proposed ratio by right lines from one angle to the opposite side.

R U L E.

Divide the opposite side in the ratio proposed, and join the angular point and points of division.

EXAMPLE. Let it be required to divide the triangle ABC , (fig. 35. plate 10.) containing $26\frac{1}{2}$ acres into three parts, in the ratio of 40, 20, and 10; by right lines proceeding from the angle C , to side AB , whose length is 28 chains.

GEOMETRICALLY.

The ratio of 40, 20, and 10, are the same as 4, 2, and 1, and their sum is 7, therefore divide A B, into 7 equal parts; and make A a equal 4, $ab = 2$, and $bB = 1$, of those parts; join C a, and C b, and its done.

ARITHMETICALLY.

Say as 7 (the sum of ratios,) is to 28 chains, (the length of A B) so is 4, 2, and 1, to 16, 8, and 4 chains respectively: therefore A a = 16 chains, $ab = 8$ chains, and $bB = 4$ chains.

If you would know how many acres there is in each part, say as the sum of the ratios is to the whole quantity of land; so is each ratio, to the quantity of acres; thus :

$$\text{as } \begin{cases} 7 : 26,5 :: 4 : 15,142857 \\ 7 : 26,5 :: 2 : 7,571428 \\ 7 : 26,5 :: 1 : 3,785714 \end{cases} \begin{matrix} = \text{triang. } A C a \\ = \text{triang. } a C b \\ = \text{triang. } b C B \end{matrix}$$

P R O B L E M IV.

It is required to divide a triangular field into any number of parts, and in any given ratio, from a given point to any side of it.

R U L E.

Divide it in the given ratio, from the angle opposite the given point, by the last problem; reduce these dividing lines to pass through the given point by prob. 19, of Geometry.

EXAMPLE. Let it be required to divide the triangular field A B C, (fig. 36.) containing 7 acres, into two parts, in the ratio of 2 to 5, from a pond *b*, in the side B C. First divide B C into 7 equal parts, make

T 3

B *a*

$Ba = 5$, then $aC = 2$; draw Aa , and it divides it in the given ratio from the angle A . Then to reduce it to the point b , draw Ab , and parallel thereto ac , join cb , and it will be the true dividing line required.

EXAMPLE II. Let the triangle ABC , (fig. 37.) be divided into three equal parts, from a gate or pond c , in the side AB ; first divide AB into 3 equal parts, as Aa , ab , bB , then the lines Ca and Cb , divides it as required from the angle A ; reduce these lines as above to cd and ce , and they will be the true dividing lines required, passing from the gate or pond c .

ARITHMETICALLY *in the field.*

Divide BC , (fig. 36.) in the ratio required, (by the last problem,) and set a
mark

mark at the point of division at a , and also at the pond b ; then go to the corner A , and with your sextant or other instrument, take the angle $b A a$, go to a , and lay off the same angle $A a c$, and it will give the point c (in the side $A C$), from whence the fence must go to the pond.

P R O B L E M V.

To divide a triangular field in any ratio required, by right lines parallel to one side and cutting the others.

R U L E.

Divide one of the cut sides in the ratio required, then find a geometrical mean proportional between the cut side and the division next the parallel side; lay off this

T 4

mean

mean proportional, and through the point draw the first parallel dividing line. Proceed in the same manner with the remaining triangle.

EXAMPLE. Let the triangle $A B C$, (fig. 38) be divided into three equal parts, by lines parallel to $A B$, and cutting the sides $A C$ and $B C$.

GEOMETRICALLY.

First divide one of the cut sides as $B C$ into 3 equal parts, set 2 of them from C to D , then find a geometrical mean proportional between $B C$ and $D C$, thus : On $B C$ continued make $C F = C D$, and bisect $B F$ in E , on the point E and distance $B E = E F$, describe the semi-circle $B N F$; erect $C N$, perpendicular to $B C$, and

B C, and it will be the mean proportional required ; make $C G = C N$, and draw G H parallel to A B, and it will cut off the first part A B G H.

Secondly, divide G C into 2 equal parts as at I, make $C K = C I$, and bisect G-K in P, on this point P, and distance P G = P K, describe the semi-circle G O K, then will C O be a mean proportional between C G and C I; make $C L = C O$, and draw L M parallel to A B or H G, and the triangle will be divided as required into three equal parts.

ARITHMETICALLY *in the field.*

Measure B C, suppose 9 chains, two-thirds of which is 6 chains = C D, then $9 \times 6 = 54$, whose square root = 7.35 chains = G C the geometrical mean ;
mea-

measure off C G or B G, and lay off G H, parallel to A B ; by measuring the angle A B C, and making the angle H G C equal to it ; this line cuts off one third part from A B. Then $\frac{1}{3}$ C G or 7,35 = 3,67 = C I, and $7,35 \times 3,67 = 26,9745$ whose root is 5,19 chains, = C L the mean proportional ; lay out L M parallel to A B, and it divides the triangle H G C, into two equal parts, and consequently the whole A B C is divided into three equal parts as required.

P R O B L E M VI.

To divide any parallelogram, viz. square rectangle, rhombus, or rhomboides, in any ratio proposed by right lines cutting two opposite parallel sides.

R U L E.

Divide the sides in the proposed ratio, and join the points of division.

This is so plain and easy that an example seems needless.

P R O B L E M VII.

To divide any right lined figure into any proposed ratio, by right lines proceeding from one angle.

R U L E.

Reduce the field into triangles and parallel trapeziums ; then divide the sides of each in the ratio proposed, and join the points of division, by problem 3d and 6th. And these lines may be straightened by problem 20, Geom.

EXAMPLE. Let it be required to divide the irregular pentagonal field A B C D E, (fig. 39.) into three equal parts, by right lines from a gate in the angle B, being
the

the only road to it. Draw $A C$, and parallel thereto $F D$, divide $A C$ and $F D$ into three equal parts in the points $a b C$, and $c d D$; join $B a c E$, and $B b d E$, which lines divide it into three equal parts. Lay a parallel ruler from E to a , and move it up to c , and it will cut the side $E D$, in e ; that is make $c e$ parallel to $a E$, and $d f$ parallel to $b E$; join $a e$, and $b f$, and the lines $B a e$, and $B b f$ divides it otherwise into three equal parts. And making $a g$ parallel to $e B$, and $b h$ parallel to $B f$, will give the points g and h , join $B g$, and $B h$, and these lines give another division into three equal parts.

N. B. I have omitted drawing the parallel lines to prevent confusion.

P R O B L E M

P R O B L E M VIII.

To divide any right lined figure in any proposed ratio, by right lines proceeding from a given point in one of its sides.

R U L E.

First divide it in the given ratio, from an angle, by problem 7th, then reduce the division from the angle to the given point by problem 4th.

EXAMPLE. Let the trapezium A B C D, (fig. 40) be divided in the ratio of 1, 2, 3, by right lines from the gate E, in the side A B, the lesser part lying to the side A D.

Draw the diagonal A C, and divide it into 6 equal parts, being the sum of the ratios ;

ratios ; make $Aa =$ one of them, and ab equal two, and bC three ; join BaD and BbD , these lines straightened from B , gives Bc and Bd , which divides it in the ratio required from the angle B . Now to reduce it to the point E , lay a parallel ruler from E to c and move it up to B and it will cut AD in e , join Ee , and it will be the first dividing line from the point E : lay the ruler from E to d , and move it up to B , and it will cut DC in f ; join Ef , and it will be the other dividing line; which lines divide it from the point E , in the ratio required.

N. B. If the lesser division must have join'd the side BC , it would be best to use the other diagonal BD .

EXAMPLE II. Let the trapezium $ABCD$, (fig. 41.) be divided into four equal

equal parts from the gate E, in the side A B, being the only road to the land.

Draw the diagonal B D, and divide it into four equal parts, join the angles A and C, with the points of division; straighten A c C and A b C, from C upon A B, and they give C f and C e; also straighten A a C, from A upon C D, and it gives A d; make A g parallel to E d, and draw E g; make e h and f i parallel to E C, and join E h and E i; then the lines E g, E h, and E i, divide it as required.

Note, It is proper to observe, that if the line A a C had been straightened from C upon A D, it would have occasioned more work; for supposing it so straightened, it gives C a, (fig. 42.) then making a c parallel to E C, and joining E c it is the diagonal of the parallel trapezium c a E C, and a c is the other diagonal of the same trapezium

trapezium ; and consequently the triangle $a E A$ is left out of the division ; therefore to bring this in, draw $a b$ parallel to $A E$, cutting $E c$ in b ; join $A b$, then straighten $A b c$ from c , and it gives the point d , make $d g$ parallel to $E c$, then will $E g$ be the true dividing line.

But it will be shorter to reduce the division from the angle C to A , thus : make $a d$ (fig. 42.) parallel to $A C$, and it gives the point d in the side $D C$, the same as in figure 41.

N. B. I have been more full and particular in this problem, because several authors have pronounced the geometrical solution impracticable ; and no one having shewn the method made use of above.

P R O B L E M

P R O B L E M IX.

To divide any right lined figure in any proposed ratio, by right lines proceeding from a given point within the said figure or field.

R U L E.

First divide it in the given ratio, either from an angle or otherwise; reduce one of the outside parts (thus divided) to the given point; then divide the remaining part a-new, by the last problem.

EXAMPLE. Let the trapezium ABCD, (fig. 43.) be divided into three equal parts, from a pond P, near the middle.

The diagonal B D, being supposed drawn and divided into three equal parts,

U

and

and straightened from the angle A ; then AE will be one of those lines, draw Pa and make ac parallel to AP , and ab parallel to PE , join Pb and Pc , then will $PbBc$ be one third part.

Now the remaining part $AcPbCD$, must be divided into two equal parts from the point P , which is easily done by the last problem thus: continue bP to F , and it reduces it into two trapeziums $AcPF$, and $FbCD$, draw the diagonals Db and AP , and divide each into two equal parts in the points d and e ; join $CdFe$, and it gives the first division into two equal parts, which being straightened upon CD , from c , gives cf , which is done thus: lay the edge of your scale from C to F , and take in your compasses the nearest distance of d from the edge of the scale, move one foot of the compasses by the edge of the scale,

scale, and the other will cut the line CD in n ; lay the scale to n and e , and take the nearest distance to F , and it will cut CD in o ; lay the scale from o to c , and take the nearest distance to e , and it will cut CD in f , join cf and it's the straightened line, and divides $AcbCD$ into two equal parts from c ; lay the scale from f to P , and take the nearest distance to c , this will cut CD in g , join Pg , and it is done.

Note, I have also been more particular in this, as no author before, that I know of, has given a geometrical solution to this problem.

P R O B L E M X.

*To divide any given circle in any given ratio,
by concentric or parallel circles.*

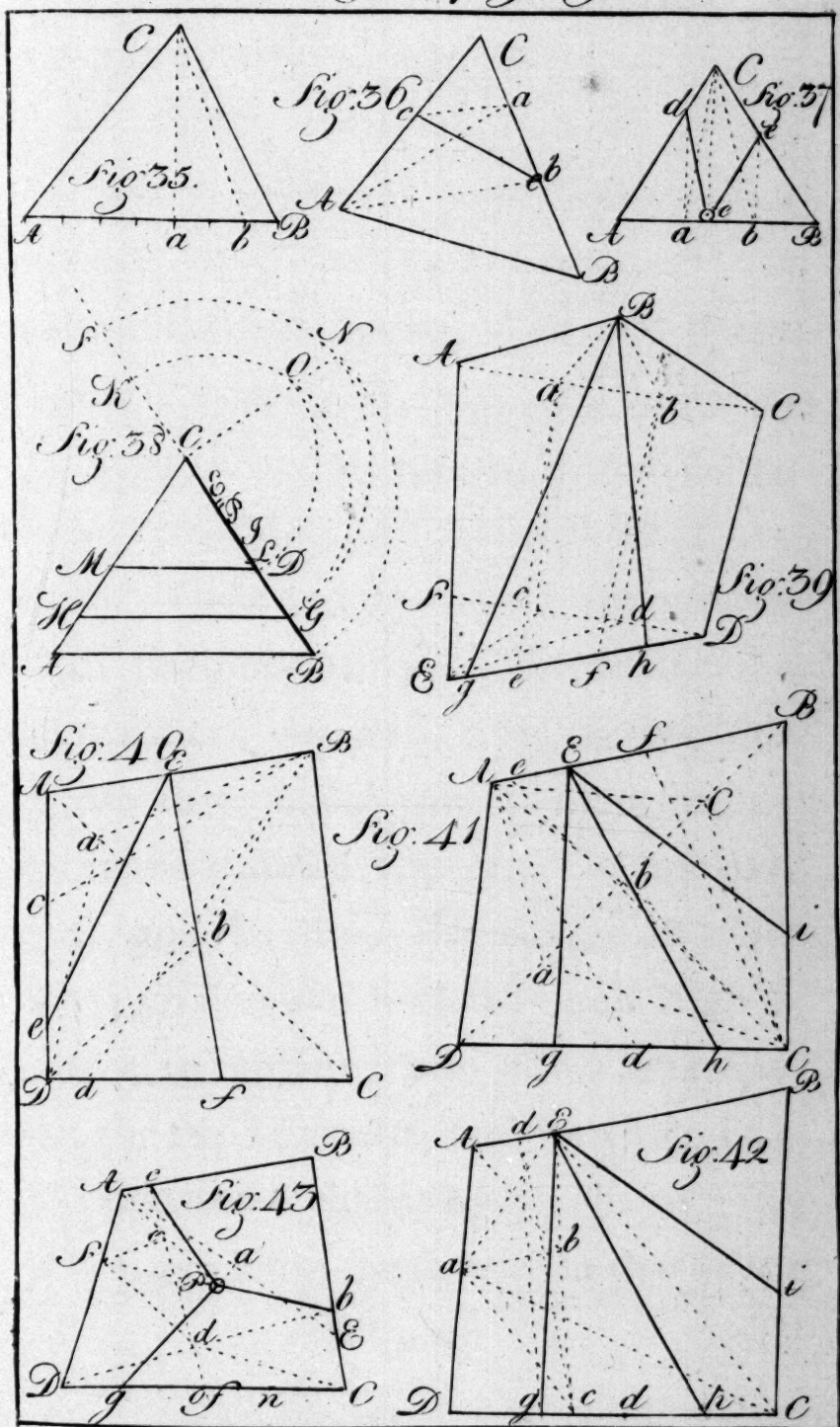
R U L E.

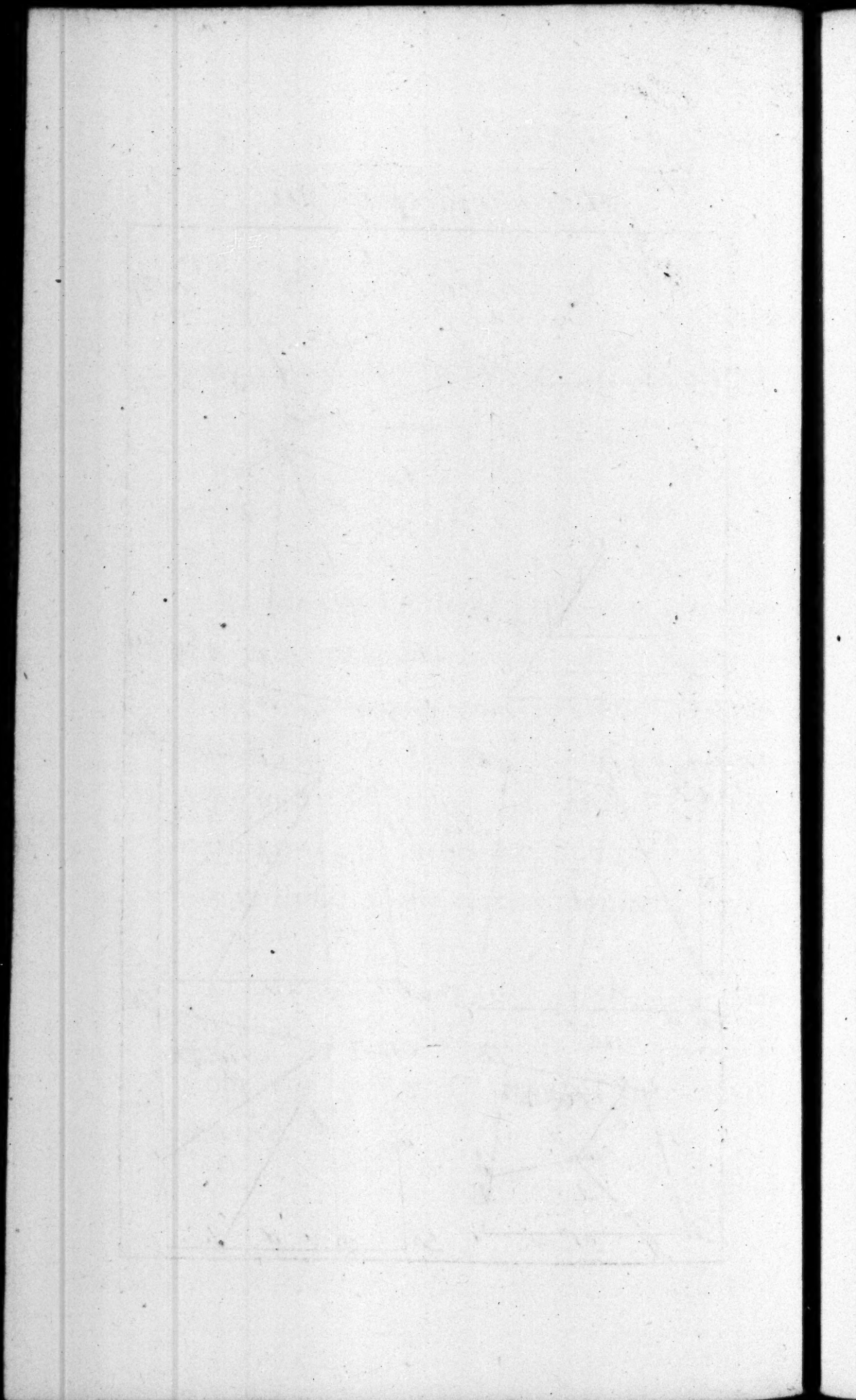
Divide the radius of the given circle in the ratio proposed; then find (by problem the 5th.) a geometrical mean proportional, between the radius and the first ratio; and also between the radius and sum of the two first ratios, &c. and they will be the radiuses of the circles required.

EXAMPLE. Let the circle (fig. 44, plate 11) be divided in the ratio of 2, 3, 7, from the center C towards the circumference.

First

Plate 10 to face page 292.





First divide the radius $A C$, into 12 equal parts; make $C a$ equal two of them, and $a b$ three, then will $b A$ be seven. Then the geometrical mean between $C a$ and $C A$, or 2 and 12 is $C e = 4,8$ the radius of the first share. And the mean between $C b$ and $C A$, or 5 and 12, gives $C f = 7,74$ the radius for the second share; and the remaining part is the third share.

N. B. The triangle $A B C$ (fig. 38.) in the 5th problem, may be divided in the same manner; by first dividing the side $B C$ into three equal parts; then finding a geometrical mean between the first part next C , and the whole side $B C$, that is between 3 chains and 9 chains, and it gives $C L = 5,19$ chains; and also a mean between the two first parts, and the whole side, that is between 6 chains and 9 chains gives $C G = 7,35$ chains the same as in that prob.

Note also, That any polygon may be divided by this problem into any given ratio, by lines parallel to its sides. For since every polygon may be considered to consist of as many triangles, as the polygon has sides; and one of these triangles divided as above, in the given ratio, gives the division of all the rest, by continuing it round parallel to each side.

P R O B L E M XI.

To divide any given circle, in any proposed ratio, by right lines from the center to the circumference.

R U L E.

Divide the circumference in the ratio proposed, join the center and points of division, and it is done.—This is so easy, that an example seems needless.

SECTION

SECTION XIII.

To measure Marl-pits.

IN *Lancashire* and the adjacent counties (says the author of the *Royal Gauger*) this sort of measuring is of great use; “for when their land has been grassed so long, till it begins to bring forth rushes, moss, &c. then they propose to marl, which they generally do in *May* or *June*, that so they may have finished before their hay harvest begins.

When the owner of the land has proposed how many acres he designs to marl, he appoints 10 or 12 labouring men

(called marlers) to come and view the same; which done, they agree upon a price, which is about 20 s. a rod or rood, for so many rods of marl that are contained in the pit. They have 8 yards long, 8 yards broad, and 1 yard deep, to the rod. (But Mr. BURNS in his treatise of surveying says, custom has established 9 yards long, 8 broad, and 1 deep, in *Cheshire* and some of the adjacent counties.) The marlers having agreed with the owners for a price, and place where the pits must be; they come in the winter and fey it, that is, mark out the shape of the pit, and dig off the earth till they come to the marl; this in some lands lieth near the surface of the earth, and in others a foot or two deep, more or less, according to the goodness of the land. The carts which are made use of in this work, are (or ought to be) 5 feet long,

long, 2,7 feet wide within, over the axle-tree, and 2 feet deep; so that a cart of these dimensions will just hold 27 solid feet, or 1 solid or cubic yard, of marl; and is accounted one load, and drawn by 2 horses from the pit to the field.

The day's work is so many thousand loads, according to the number of hands employed.

They begin in the morning at three o'clock or before, and leave work about three in the afternoon. They have two men in the pit called feyers, and the rest fillers; and in the field there is a man (or two if the work is large) called a setter, that is, directs the driver of the cart where to empty his load, and there the setter spreads the marl thin upon the ground, and thus it lies till the next spring, when they plow it up and sow it with oats. This way

way of manuring the land is counted the best, because it will endure many years, without any other repair.

When the marlers have finished, they set a day apart to dress the pit, which they do with flowers, and the drivers and horses are dressed with posies or nosegays, at which time there is a good feast provided, with a fine posie or garland dressed with plate, rings, watches, &c. (after the manner of milk-maids on a May-day in *London*) prepared by some of the country maids, which they bring to the pit, and present it to the young man that drives the master's team, he having the honour all that day to hold the posie or garland; which is reckoned amongst them a very great piece of honour."

After this the workmen and master, agree to make choice of some able surveyor to
measure

measure the pit, which is done by the following

R U L E.

Reduce the surface of the pit into trapeziums and triangles, by the help of some good packthread, then measure the bases and perpendiculars, in feet and decimal parts; and also take several depths in the most convenient places (as your own judgment will best direct) in the like measures, in order to obtain a mean depth. Find the area of the whole surface, and multiply it by the mean depth, gives the content of the whole in feet; divide this by the feet in a rod, and the quotient will be the answer in rods and decimal parts.

EXAMPLE. Suppose figure 45, plate 11, represent the eye-draught of the top surface-

300 OF MEASURING SEC. XIII.

face of a marl-pit, whose dimensions in feet and decimals stand thus;

Depths.		C H	A E	F G
1	4,2	30,5	70	35,5
2	6,8			
3	9,4			
4	8,7	B K	A C	
5	10,6	4	42,5	
6	11,3			
7	14,7			
8	16,8	D I	C E	
9	15,8	5,2	31,5	
Sum	98,3			

Now 98,3 the sum of the depths divided by 9 the number taken, gives 10,92 for the mean depth. And the area of the trapezium A C E F = 2310, and the triangle A C B = 85, also the triangle C E D = 81,9, and their sum = 2476,9 the whole area; which multiplied by 10,92 the mean depth gives 27047,748 the content

tent in solid feet. And this divided by 27, the solid feet in a load, gives 1002 loads; and also divided by 1728, the solid feet in a rod, gives 15,652 rods, which at twenty shillings a rod, amounts to 15*l.* 13*s.* 0 $\frac{1}{2}$ *d.*

But according to BURNS, the solid feet in a rod are 1944, consequently the pit contains but 13,907 rods, and it's amount but 13*l.* 18*s.* 1 $\frac{1}{4}$ *d.*

Remark. The above method of finding a mean depth, is subject to very considerable errors, not only in the measuring of marl-pits, but in many other parts of mensuration. And therefore to obtain a true mean depth, it is best to proceed thus: take the depth at proper places on both diagonals, noting down at what distance they are taken from the end of the diagonal

gonal (in the same manner as taking off-sets on the station lines) then protract the dimensions of each diagonal, and find a mean depth geometrically, in the same manner as directed to find a mean off-set, (see the remark at the end of the first example, sect. 8.) add together the mean depths so found on each diagonal, and take half their sum, will give a correct mean depth.

EXAMPLE. Suppose the dimensions taken upon the diagonal A E, as follows:

Length on diagonal	Depths.
0	4,2
20	8,7
40	10,6
60	14,7
70	16,8

These

These dimensions protracted from a large scale will give $A B G E D C$, (fig. 46.) in which $A B$ represents the diagonal line, and $C D E F G$ the bottom of the pit under the diagonal; $A C$, $a D$, $b E$, $c F$, and $B G$, the different depths, which straightened gives $A H$, bisect $B H$ in I , so shall $B I$ be the mean depth on the diagonal $A E$ (fig. 45.) = 10,5 feet. But the sum of the depths on this line divided by their number, viz. 5, gives 11 feet for the mean.

Proceed in the same manner with the other diagonal.

N. B. If more exactness is required, find the area of $A B G F E D C$, (fig. 46.) arithmetically, as if it was an off-set; divide this area by the length of $A B$, and the quotient will be equal $B I$, the mean depth.

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SECTION XIV.

*Of Leveling, for the conducting of water, either
in pipes or canals.*

LLEVELING is the finding a line parallel to the horizon, at one or more stations, and so to determine the height of one place with regard to another.

A truly level surface is a segment of any spherical surface which is concentric to the globe of the earth ; and of such is the sea.

X

A true

A true line of level is an arch of a great circle which is imagined to be described upon a truly level surface.

The apparent level is a straight line drawn tangent to an arch or line of true level. Thus let R A H, (fig. 47, plate 11.) be the arch of a great circle of the earth, C its centre, R H the rational horizon to a person at A; then B A D is the line of apparent level, being a tangent to the arch E A F, of true level; and also parallel to the rational horizon R H.

Every point of the apparent level, except the point of contact, is higher than the true level; for B is farther from C than A is, and I is farther than B, &c. therefore the greater the distance from A, the more the apparent level differs from the true.

If

If two points E and F be equally distant from A, then will the difference of the true and apparent level be equal at each place, that is $BF = DE$; but if the two points be at unequal distances from A, as E and I, then will the difference of true and apparent level be different; for IG is greater than DE. And therefore in such cases it is necessary to know what the difference between the true and apparent level is for any distance from A, and may be found thus :

Suppose the distance of AB be one mile, then in the right angled triangle BAC, is given $BA = 1$ mile, and $AC = 4000$ miles = the earth's semi-diameter, to find BC: now (per. 47. Eu. 1.) the square of $AC +$ square of $AB =$ square $BC = 16000001$ whose root is $4000,000125 = BC$, and AC or FC taken from BC, leaves

X 2 BF, the

B F, the difference of level at B, equal 0,000125 parts of a mile = 7,92 inches, or 8 inches very nearly. Therefore the apparent level exceeds or is higher than the true level by 8 inches = B F, at the distance of one mile from A; and as A C is constant, it will be as the square of A B is to F B, so is the square of A I to G I, &c. and by taking A B = 80 chains the following table was calculated, shewing the difference of true and apparent level from 5 to 100 chains distance.

It may be proper to observe here, that if a canal of a mile or 80 chains in length was to be cut from A to B, that the bottom of the canal must be a curve line deviating all the way from A to F, from the apparent level; where F must be sunk 8 inches lower than B, and so in proportion to the distance of the levelling instrument
at

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at A ; as may be seen in the following table.

Dist. in Chains.	Dif. lev. Sub. In.	Dist. in Chains.	Dif. lev. Sub. In.
5	0,03	55	3,78
10	0,12	60	4,5
15	0,28	65	5,31
20	0,5	70	6,12
25	0,78	75	7,03
30	1,12	80	8,
35	1,53	85	9,03
40	2,	90	10,12
45	2,53	95	11,28
50	3,12	100	12,5

This I thought necessary to premise before I entered upon the description of the instruments made use of, the method of adjusting them, and the practice of leveling.

The necessary instruments are a level, of which there are several sorts; two station slaves, and a chain.

Of the L E V E L.

There are many sorts of levels, but I shall confine my description to one of the best sort (see fig. 48.) which consists of a brass achromatic telescope A, 6, 9, or 12 inches long, shewing objects in an erect position, and taking in as large a field of view as possible; in the focus of the glass next the eye when used, is fixed an horizontal hair to cut the object in time of observation: the glasses should all be fixed in cells, and these cells fixed by screws, and not to be taken out as long as can be avoided: the object glass should be fixed in a short tube, sliding within the other tube to have a small motion for adjusting the focus of distinct vision to all distances, by the help of a screw, in the same manner

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ner as the small speculum of a reflecting telescope is moved.

Upon the telescope is fixed a small brass tube B, within which is a glass tube, filled nearly full with spirits of wine or oil of tartar, these not being liable to freeze; and hermetically sealed at each end, and the brass tube is filed away on the upper side, so that the glass tube and the bubble of air moving from end to end may be seen, which bubble is the direction for setting the instrument. The bore of the tube and length of the bubble should be greater according to the degree of accuracy required; for the larger these are, the friction will be less, and the motion more sensible.

The telescope is fixed upon a brass bar, underneath which is a socket for fixing it to the adjusting plates C, and three-legged

staff; the bubble is readily brought to rest in the middle of its tube, by means of the four screws in the adjusting plates.

Of the Station Staves,

In the practice of levelling we must have two station staves each ten feet long, and divided into feet, inches, and tenths, and figured accordingly.

Upon each staff is a vane made to slide up and down, and also to stand against any division on the staff by the help of a spring. These vanes may be three or four inches wide, and ten inches long; they may be divided (by 2 lines) into three equal spaces, and let the two extreme spaces be painted white; and let the middle space be divided also into three smaller equal spaces, and let that in the middle
be

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be painted white, the other two black, which will render them fit for all distances.

I shall say nothing of the chain here, as it has been explained in the foregoing part of this work.

Being thus provided with a level, two station staves, a chain and two assistants we may proceed to the work ; but first it will be necessary to try whether or no the level be well adjusted,

Of adjusting the Level.

Choose a large field or meadow, that is nearly level, and set down the instrument about the middle ; from which measure out a right line of any convenient length, as suppose 20 chains from A to B, (fig. 49.) and in the same direction measure 20 chains

chains more from A to C, so shall B C be 40 chains distant; let one assistant with his staff be left at C, and the other go to B, whilst you return to the instrument at A, and adjust the telescope at right angles to the line B C, so as the bubble may rest in the middle or mark in its tube; then turn the telescope towards your assistant at C, and make the bubble rest in the middle, and observe where the hair in the telescope cuts the staff, and direct your assistant to move the vane up or down till the hair cuts the middle thereof.

In like manner direct the telescope to the other assistant at B; and let the vane be placed so that the hair in the telescope cut the middle of it, when the bubble rests in the middle of its tube, so are the vanes placed in a true level or horizontal plane.

Remove

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Remove the instrument to that assistant next the sun, that you may have the advantage of its rays (if it shines) upon the other assistant's vane, and there set down the instrument as near the staff as you can; then having set the instrument horizontal, so that the bubble rests in the middle of its tube; observe what divisions on the staff above or below the middle of the vane, are even with the axis of the telescope; for so many divisions must the other assistant's vane be raised or depressed, which direct him to do accordingly: as suppose the instrument be moved to B, and when adjusted, the axis or center of the telescope is 1 foot 6 inches above the middle of the vane at B, then must the vane at C be raised 1 foot 6 inches to make it in a true level with the axis of the telescope.

But

But as the instrument must shew the apparent level, and this varying all the way from B to C from the true level, being so much higher as the distance is greater; therefore at the distance of C from B, viz. 40 chains, it is by the foregoing table 2 inches higher; consequently the vane at C must be raised 2 inches above the true level of the centre of the telescope.

Now direct the telescope to the vane thus raised, and if the hair cuts the middle thereof, while the bubble rests in the middle of its tube, the instrument is right; but if not, you must raise or depress the telescope (by the adjusting plates) till the hair cuts the middle of the vane; and then by the help of the screws that fix the level tube to the telescope, make the bubble rest in the middle of its said tube; so is the level adjusted.

And

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And now you may proceed to the practice of levelling, for the conducting of water from one place to another.

EXAMPLE I. Let it be required to know if water can be brought from a spring at A to a house at B, (fig. 50.) either by pipes or a canal, &c. First plant the instrument at any convenient distance from the spring, as 100 or 200 yards, or such a distance as you can conveniently see the station staves, one being placed by an assistant at the spring, and the other about the same distance from the instrument towards B, as at station 1. Direct the telescope (over two screws of the adjusting plates) at right angles to the line A B, and adjust the bubble; then direct the telescope to the staff *b*, and adjust the bubble, and instruct your assistant to place the vane so as when the
staff

staff is held perpendicular the hair in the telescope may cut the middle of the vane; enter in the field-book under back-sight, the feet, inches and decimal parts, cut by the lower edge of the vane, viz. 4 feet, 8,4 inches, (see the following field-book) turn the telescope towards *f*, and adjust the bubble, and direct your assistant there, to place the vane so as when the staff is held perpendicular the hair in the telescope may cut the middle of the vane; then enter the divisions cut by the lower edge of the vane in your field-book under fore-sight, suppose 5 feet, 10,6 inches.

Now place the instrument at station 2, and let the staff at *f* remain there still, but let an assistant carry the staff that was at *b*, at the spring, forwards to *f* in station 2; adjust the cross level, that is, make the bubble rest in the middle of its tube, at a
right

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right angle to A B; then turn the telescope to the remaining staff *f* now marked *b*, and enter the numbers cut by the vane there, under back-sight, viz. 4 feet 6,2 inches, then look forwards to *f* in station 2, and enter the observation under fore-sight, viz. 3 feet, 1,2 inches: do the like when the instrument is placed at stations 3, 4, 5, &c. always taking care to keep the staff in the same place when you looked at it for a fore-sight, till you have also taken with it at a back-sight; as fig. 50, will better explain than a multitude of words.

Being arrived at B, and having finished your level, add up the column of back-sights into one sum, and the column of fore-sights also into one sum; and the difference between these sums, is the difference of level between A and B; and if
the

the sum of the fore-sights is greater than the sum of the back-sights, then B is lower than A; but if the sum of the fore-sights is less than the sum of the back-sights, B is higher than A. See the field-book.

FIELD-BOOK to EXAMPLE I.

Back-sights.			Fore-sights.		
Stat.	F.	I.	Stat.	F.	I.
1	4	: 8,4	1	5	: 10,6
2	4	: 6,2	2	3	: 1,2
3	2	: 3,1	3	4	: 10,5
4	8	: 11,2	4	4	: 6,2
5	2	: 11,6	5	4	: 2,4
6	3	: 6,5	6	5	: 0,6
7	6	: 1,2	7	7	: 4,3
8	4	: 10,5	8	5	: 6,2
37 : 10,7			40 : 6,0		

here the sum of the fore-sights is greater by 2 feet, 7,3 inches, therefore B is lower than A so much; and if the distance from A to B be not more than 5 miles, the water may be brought very well; for experience

rience shews, that water will run with a fall of 6 inches in a mile, nay some assert it will run with a fall of $4\frac{1}{2}$ inches in a mile.

Note. In this example the instrument is supposed to be placed nearly in the middle between the station staves, and the distances small, so that there need no allowance for the difference of true and apparent level; or the earth's curvature, as it is commonly called. But in the next example it will be necessary to make use of the allowance as the foregoing table directs.

EXAMPLE II. Where it is required to find if water can be brought from a well on the side of the hill at A to the house at B, (fig. 51).

Y

Begin

Begin at the well A and measure towards B till you come to the brow of the hill at station 1: suppose 30 chains, which enter in the column of distance, under back-sights, (see the following field-book) and in the next column, against it enter the difference between the true and apparent level taken from the foregoing table, for the distance of 30 chains, viz. 1, 12 inches, there place your level; and as there is a large valley, and a sight can be taken to the other side; let one of your assistants go over with his staff, and the other return to the well, whilst you prepare the level; which done, take your back-sight (as directed in the foregoing example); let that assistant bring his staff without altering the vane, while you make the observation to the fore-sight; enter the divisions cut by the lower edge of the back-sight vane, in
the

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the second column of back-sights. Take up your level, and measure on to your foremost assistant at *f*, suppose 60 chains, which enter under dist. in fore-sights, and against it the dif. of level, viz. 4,5 inches; also enter the divisions cut by the lower edge of the vane 9 feet 2 inches; then measure on to station 2, suppose 20 chains, which enter, and also the difference of level for that distance, viz. 0,5 inches, let one assistant go on to *B*, the other return to *b* in station 2, while you place the instrument level; then take the back-sight, suppose 0 feet 6,5 inches, then the fore-sight, and measure on to *f* in station 2, suppose 50 chains, which enter, and also 3,12 inches the difference of level for that distance, and then the divisions cut by the vane, viz. 8 feet 11,2 inches.

And thus may you proceed though the stations were ever so many, and the distance of A from B ever so great.

Add up all the columns except the stations, then from the sum of the heights under back-sights, take the sum of the differences of level under the same; the remainder will be the corrected sum of the heights of the vane: do the same under fore-sights; and the difference of these corrected heights, gives the difference of true level; and the sum of the distances under both sights will be the whole distance also from A to B.

FIELD.

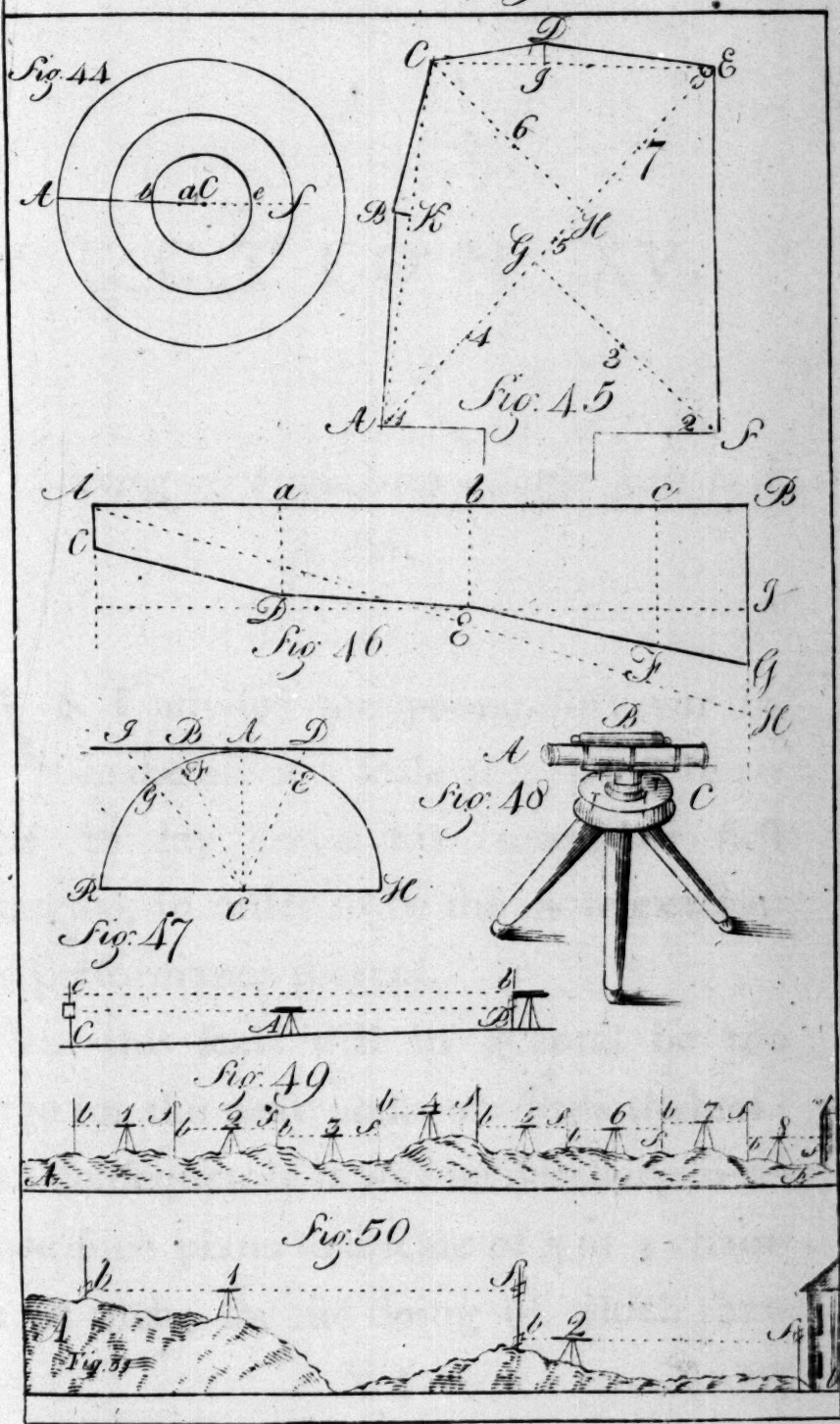
FIELD-BOOK to EXAMPLE II.

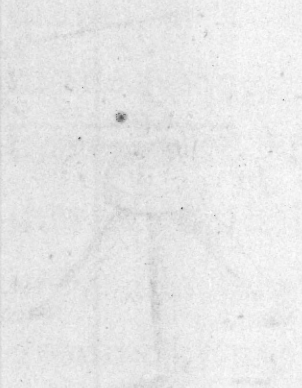
Back-fights.				Fore-fights.			
St.	Ht. v. F. In.	Dft Ch.	Dif. lev. In.	St.	Ht. v. F. In.	Dft Ch.	Dif. lev. In.
1	0 : 3,5	30	1,12	1	9 : 2	60	4,5
2	0 : 6,2	20	0,5	2	8 : 11,2	50	3,12
	0 : 9,7	50	1,62	18.	1,2	110	7,62
	1,62				7,62	50	
	0 : 8,08			17	5,58	160	
				0	8,08		
				16	9,5		

Here the sum of the heights of the vane in the fore-fights, exceed those of the back-fights by 16 feet $9\frac{1}{2}$ inches; and so much is B below the true level of A; and the whole distance being 160 chains, which divided by 80, the chains in a mile, quotes 2 miles: therefore the fall is 8 feet $4\frac{3}{4}$ inches in a mile.

Note. When water is to be brought in pipes, they may be laid near the surface of the earth, down through vallies, and over the risings, provided they be not above the level of the surface of the spring, for should the fall be very great from A to B, yet if there be a rising in the way between them, higher than the surface of the spring or well; the pipes must be laid so much deeper, so as to keep them below the level of the spring. But if the water is to be brought in an open canal, the bottom thereof must be carried on nearly level, only allowing for the fall; and if the ground be up hill and down, it must be sunk or raised accordingly, unless you can carry on the level by going round about.

Plate 11 to face page 526





SECTION XV.

Of Reducing of Plans from a large scale to a smaller.

AS I advised the young surveyor to make use of a scale of 2 chains in an inch, to lay down his rough or first draughts, in order to be the more exact in the performance thereof.

But this scale will in general be too large for the neat plans or maps thereof; and consequently it will be necessary to reduce such plans to a scale of 3 or 4 chains in an inch; for the doing of which there

are several methods made use of, or recommended by different authors.

But I shall mention only two; the first is by similar squares, and the second by the pentagraph or parallelogram.

To reduce plots by similar squares.

Let fig. 52, plate 12, represent a plot of four inclosures laid down by a scale of 2 chains in an inch, and it is required to reduce it to half its linear dimensions; that is to a scale of 4 chains in an inch.

Draw a line nearly across the middle, with a black lead pencil, as A B, cross it near the middle with another line as C D, and at right angles to the former; from the point of intersection o, lay off the equal divisions 1, 2, 3, &c. each way on the lines A B and C D, and through the points

points of division draw pencil lines parallel to *A B* and *C D*, and they will form a number of little squares.

Then upon the paper or vellum designed for the new plot, draw two lines nearly across the middle and at right angles to each other, with a pencil; from the point of intersection lay off the equal divisions 1, 2, 3, &c. each way on the lines *a b* and *c d*, (fig. 53.) each division here must be but half the length of those on the old plot (fig. 52.) through these divisions draw pencil lines parallel to *a b* and *c d*; and you will have the same number of little squares (only half the length) as in the old plot.

Now observing what squares and part of such squares, the bounding lines of each piece or inclosure pass through in the old plot; and make the same lines pass through

through the same squares, &c. in the new plot; (see fig. 53.) and you will have a similar plot of four pieces, and of half the linear dimensions of the old one: and that occupies but one fourth part of its surface.

And in the same manner may any number of pieces be reduced; and in any proposed ratio; for it is only making the divisions on the lines *a b* and *c d* in the given ratio to the divisions on the lines *A B* and *C D*.

Note. When the sides of the inclosures are drawn in the new plot, the pencil lines may be rubbed out with a bit of stale bread.

A description of the Pantagraph.

Before I proceed to the second method of reducing plots, viz. by the pantagraph,
it

it may not be improper to give a short description of that instrument; and how those that live in the country, and may not have an opportunity of purchasing one, may construct one for their own use.

There are different sorts, some very complex; but I shall only mention one of my own contriving; which I think is not only the most simple, but easy and exact in its performance. It consists of four strips or rulers of deal or any other light wood; the longest of which A D, (see fig. 54.) is 39 inches, and F D 27 inches; also G C and G E, are each 15 inches, and all of them $1\frac{3}{4}$ inches broad and $\frac{1}{4}$ of an inch thick.

Having procured these rulers, draw a fine line down the middle of each; then divide the ruler A D thus, make a prick with your protracting pin at A on the
middle

middle line, an inch and half from the end ; lay the edge of your plotting scale to the line down the middle, with o on the point A, and prick off 12 inches from A to B, do the same from B to C, and from C to D ; also prick off the same distance on the rulers G C and G E ; and on the ruler F D, from F to E, and from E to D ; also prick off 6 inches from F or E to H ; and with a center-bit fixed in each prick, bore holes of 4 or 5 tenths of an inch diameter ; and get 6 pins turned of some hard wood, and exactly to fit the holes ; the two at C and E may be short ones, with knobs on the top, and fastened with wires through them on the under side ; the other four should be near three inches in the under part, and turned with large shoulderings to keep them steady in the holes, and fastened above with wires
close

close to the rulers : that at D should have a knob at the bottom : that at F called the fixer, should have a piece of a strong needle or pointed wire fixed in the lower end to stick into a table when used : that at G called the copier should have a hole through the center or axis to stick in a pencil ; and that at B turned tapering to a point, which is called the tracer.

Now from this construction it will plainly appear, that in any position or opening of the instrument, C D is equal and parallel to G E ; and E D is equal and parallel to G C, whence (I suppose) the name of parallelogram.

Let us suppose the instrument fixed on a level table by the fixer F, and the tracer B to be moved either along a straight or curved line to *b*, then will the copier or pencil G, describe a line G *g* exactly similar,

lar, but only of half the length; for as it is now fixed, G will always be in the middle between F and B; that is, $FG = GB$, and $Fg = gb$; therefore as $FB : GF :: Fb : Fg$; also as $FB : FG :: Bb : Gg$, but FG is = half FB , consequently Gg is = half Bb .

Also observe, the points F, G, and B, will always be in a right line; and so will the points A, G, and H; for when B is brought to b , D will slide on its knob to d , and C to c , and the other points to the corresponding small letters; and if the right lines bF and ah be drawn, they will both pass through the point g .

From the above construction and theory, the use of this instrument will become very plain and easy; especially when the rough plot is small enough to be copied off at one operation.

EXAM-

EXAMPLE I. Suppose it is required to reduce the plot of the same four inclosures as fig. 52, which were laid down by a scale of 2 chains in an inch; to one of 4 chains in an inch, or to half its linear dimensions.

Fix the clean paper and rough plot beside each other, upon a level table with small nails, (see fig. 55 and 56.) then fix the pantagraph on the table, by sticking in its fixer as at *f*, in such place, as when the tracer *b* is moved over the rough plot, the copier may move on the clean paper; then beginning at one corner with the tracer, move it gently along the lines round the rough plot, and the copier *g* will draw the similar figure of half the length in the lines; lift up the tracer and copier a little (taking care not to shift the fixer) and bring the tracer to the cross lines,

lines, moving it along them all one by one, so will the copier draw similar lines on the blank paper; and thus you easily obtain an exact reduced copy, in the ratio of 4 to 2, or 2 to 1.

N. B. In the above example the tracer has moved over all the lines but one, and consequently the copier has copied all but one; then moving the tracer *b* down the remaining line, and the copier *g* will draw the same on the reduced plot.

EXAMPLE II. Let it be required to reduce the same plot two-thirds only, or in the ratio of 3 to 2; that is from a scale of 2 to 3 chains in an inch.

To perform this, the fixer must be put in the hole at A, and the tracer in the hole at H; then proceed as in the foregoing example.

example. For it will be as $AH : AG :: 3 : 2$. (see fig. 54.)

EXAMPLE III. Let the same plot be reduced in the ratio of 6 to 2, or 3 to 1; that is from a scale of 2 to 6 chains in an inch.

In this case, the fixer must be put in the hole at H, and the tracer in the hole at A; then proceed as above, For now, as $AH : GH :: 3 : 1$,

EXAMPLE IV. Let the same plot be copied off in the same bigness. Put the fixer in the hole G, and the tracer and copier in the holes F and B. Then is $FG = GB$; therefore the plan and copy will be equal. And if while the fixer remains at G, the tracer and copier be alternately placed at A and H, we shall have

Z AG :

$A G : G H :: 2 : 1$; and as $H G : A G :: 1 : 2$, and in the same ratios will the plot be to the copy.

N. B. If the copies already obtained be still too large, they may be reduced over again, in any of the foregoing ratios; and thus you may obtain copies as small as you please.

Also by changing the places of the tracer and copier, in the foregoing examples, you may enlarge your copy in the same ratios.

How to proceed when your plot is so large that you cannot copy it off at one fixing of the pantagraph.

Although this is generally the case, yet I do not find one author that has taken the least notice of it.

If

If your plot contains several hundred or thousand acres, it cannot be copied at one fixing of the pantagraph; therefore in such (and indeed in all) cases it is best to copy your plot on cartridge paper, in the proposed ratio you intend your neat map; and then copy it off again from this, on your vellum, by the directions given in the next section.

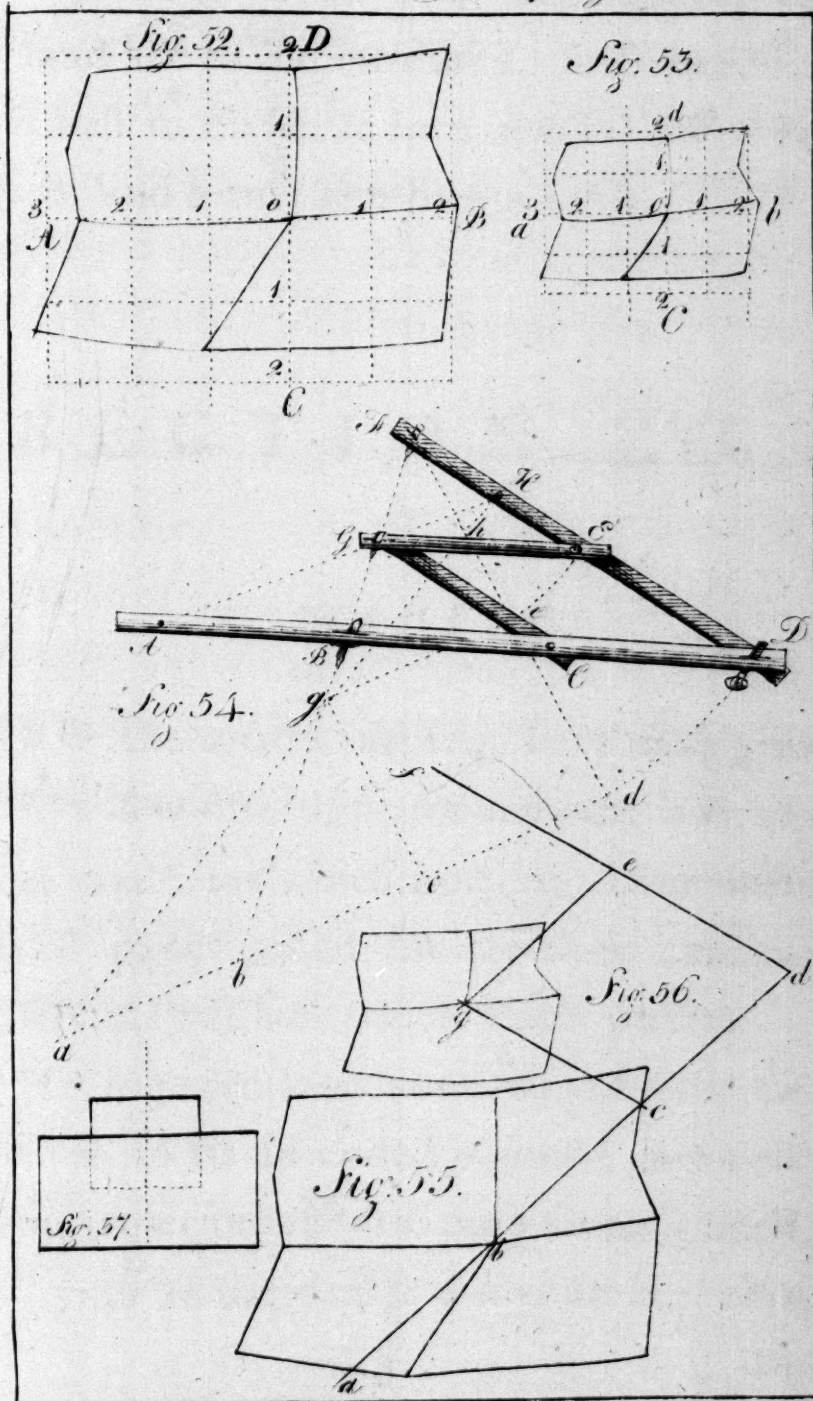
Fix your blank paper on a level table, if large enough, or else on a level boarded floor; then draw a pencil line across the middle of it, continuing it a little on the table or floor; and also draw a corresponding line across your plot; then fix it over your blank paper (except so much as you can copy off at one set) in such manner, that the two lines just mentioned may coincide, or that on the plot lie exactly over that on the copy, (see fig. 57.) this

done, fix your pantagraph in the continued line on the table, and copy off as much as you can. Remove the pantagraph, and fix it nearer the plot, but still in the same line; also unfasten your plot, and double under the part copied off; then fix it again in such manner, as to copy as much more as you can, but particular care must be taken to fix it so, that the cross lines may exactly coincide, and the tracer being moved over some of the lines on the plot which are already copied off, the copier may exactly trace the same lines on the copy.

And thus by shifting and doubling under still, what is copied off; you may copy any plot, though containing several thousand acres; and in any of the foregoing ratios with the greatest exactness.

N. B. But

Plate 12 to face page 340



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a

N. B. But when such large quantities of land are to be measured, and mapped; it is best to divide it into several different maps, and form them into a book.

S E C T I O N XVI.

Of M A P I N G.

BY the article mapping, I mean to give a few directions for drawing out the neat map; for which purpose, some make use of paper pasted on cloth or canvas; others use vellum, which I also prefer.

Having reduced your plot by the last section to its intended bigness; lay your clean paper or vellum on a smooth table, and your reduced plot a-top on it; then

Z 3

lay

lay weights or books on the top of them, while you with your protracting pin prick through every corner, bend, gate, stile, house, pool, river, &c. on your plot; these pricks or points will plainly appear on the vellum, then with a black lead pencil trace out the lines from point to point, turning up your plot by little at a time, and compare it and the copy as you go on. If any mistake should happen, take care to correct it. And thus with care you may easily and expeditiously copy off any plot of the same bigness, though it contains ever so much land.

Having transferred your plot to the vellum, take it off, and trace over all the pencil lines with a stiff fine nibed pen, and good India ink. Make the representation of gates, stiles, hedges, bushes, trees, foot and cart roads, (making a single dotted
line

line for a foot, and a double one for a cart road) pools, rivers, houses, &c. in their proper places; observing to place the tops of the houses, trees, gates, &c. all one way, which is generally towards the north. Let the houses, &c. occupy no more space than their ground plot. If the hedges, trees, houses, rivers, pools, &c. be neatly shaded with India ink; it will add greatly to the beauty of the map. In the middle of each inclosure, you may write its name and content, in acres, roods, and perches; or else a number referring to a table of names and contents. Likewise in a vacant part insert the title, neatly wrote within a compartment, properly ornamented; and also in another place add the scale suitable to the map, with the points of the compass shewing the situation of the land, and other remarkable objects

But to do all this well, the surveyor ought to be well acquainted with the art of drawing.

N. B. Finish the whole of your neat plan with good Indian ink.

APPENDIX

A P P E N D I X.

CONTAINING THE THEORY OF THE CATOPTRIC SEXTANT:

*And some further Uses in taking HEIGHTS,
DISTANCES, &c. and in measuring STAND-
ING TIMBER, &c. With the Use of the
SLIDING RULE in casting up LAND, &c.*

S E C T I O N I.

Of the Theory of the Sextant.

1. **S**UPPOSE G fig. 1. (Appen.) be a
plane, mirror, or piece of a looking-
glafs ;

glass, and an object be placed directly before it as at A, it will appear by reflection to an eye at A, as far behind the glass, and in the same direction as at B.

2. If an object be placed at C, and a ray issuing from it fall obliquely upon the glass G, it will be reflected in the direction G D; and an eye at D will observe the image of C, at E, as far behind the glass as C is before it.

3. And supposing A G perpendicular to the plane of the glass, then the angle A G C is called the angle of incidence; and the angle A G D the angle of reflection: and these two angles are in all cases equal each other; that is the angle of reflection is ever equal the angle of incidence; and upon this property of catoptrics depends the whole theory of the sextant.

4. Sup-

4. Suppose $A B M$ fig. 2, (Appen.) be a sextant, A the index glass, and C the horizon glass, $A B$ the direction of the index when it stands at 0, or the beginning of the divisions, and $A P$ perpendicular thereto; then an object or ray falling on the index glass in the direction of the horizon $H A$ will be reflected in the direction $A D$, making the angle $P A D$ equal the angle $P A H$ (by 3), but the horizon glass being placed at C , and parallel to the index glass A , it will obstruct the reflected ray $A D$, and reflect it back in the direction $C I$, parallel to $H A$ or the horizon; for the angle $H A C$ equal the angle $A C I$; therefore the object seen through the unsilvered part of the horizon glass, and its image seen in the silvered part, will appear to coincide, or both be seen in one and the same place, by an eye at the sight vane

vane at I; and in this case the instrument is said to be adjusted.

5. Again, suppose an incident ray SA from the sun or star S , (or from any other object) falls on the index glass in the direction SA , (fig. 3.) and it is required to find the motion and position of the index and glass, to reflect the image of the same to the horizon HA .

6. Now in order to do this, the index glass must be moved into such a position, as to reflect the incident ray SA to the horizon glass C , making the sum of the angles of incidence and reflection equal to the angle SAC ; therefore bisect the angle SAC in p , and draw pA , and perpendicular thereto draw FE , and it will be the position of the index glass; and also of the index likewise; and shews that the motion of the index is equal
the

the angle $B A E$, equal half the angle $S A G$.

7. For supposing $P A$ perpendicular to the index glass when the index is at o , and $p L$ be made equal $P K$; then will the angle $K A L$ equal the angle $L A D$, equal half the angle $S A G$: but the angle $L A K$ is equal the angle $p A P$ (by construction) and also equal the angle $B A E$, the motion of the index.

8. And as this motion is but half the measured angle $S A G$, is the reason why the sextant or sixth part of a circle is divided into 120 equal parts, called degrees; but in fact are only half degrees.

9. If the instrument be properly constructed, so that the visual line $I C$ may pass close by the index glass, and not, as is generally the case, at the distance of one third or more of the instrument's radius ;
then

then when the horizon glass is parallel to the index glass, an object and its image will appear to coincide not only at the distance of half a mile or more, but even at the distance of 20 or 30 yards or less.

10. But this will not be the case in the common construction, where the visual line I C is 6 inches or more from the center of the index glass; for if the instrument be adjusted by an object at the distance of half a mile or more, and then it be compared with an object at the distance of 20 or 30 yards, the image will not coincide, but fall or appear a little to the left of the object: and so much will be the error of the angle measured between any two near objects; and the nearer the object taken directly (or that view'd through the horizon glass) the greater the error.

11. For let it be required to measure the angle DAG (fig. 3.) now supposed an horizontal angle, and D and G two near land objects; and G is the object viewed directly. Also suppose K and H are two other objects in the same right line AG , but at different distances from A ; then if the image of the object at D , be brought to coincide with the object at G , it will appear to the left or beyond the object at K , and to the right or short of that at H , from an eye at I ; notwithstanding D makes exactly the same angle with all three, from the center of the index glass at A ; and the error of the angle between D and G , is equal the angle IGA ; also the error at K is the angle IKA ; but the angle IKA is greater than IGA or IHA , and consequently the nearer the direct object, the greater the

the error; and also the farther the visual line I C is from the center of the index glass, the more is the error increased.

12. I shall next observe that a very peculiar circumstance attends this instrument, and that is, that the error I have just been speaking of, may be corrected at the time of taking the angle.

13. For if the sextant be adjusted to the near direct object at G, by setting the index at 0, and looking at the direct object; and if its image does not coincide, move the horizon glass round a little on its center, till they do exactly coincide; then the image of D being brought to coincide with G, the index will shew the true measure of the angle D A G.

14. Or if the sextant has been already adjusted by a distant object; and when compared with the near direct object, the
image

image does not coincide ; move the index a little back till they do exactly coincide ; and then the minutes contained between o on the index, and o on the limb of the sextant, will be the angle of error $I G A$; and in such case is always to be added to the angle shewn by the index when the reflected image of any object at D is brought to coincide with the direct object at G ; and their sum will be the true angle $D A G$.

15. From the foregoing theory, it is easy to observe another very material fault in the common construction of the sea octant, or more especially the sextant ; and that is by placing the horizon glass and sight vane so far from the line $A H$, (fig. 3.) that it makes the angle $H A C$ very considerable ; and the ray $A H$ has likewise a considerable angle of incidence

$A a$

$H A P,$

H A P, when no angle is measured, but when the index stands at 0: and consequently when any angle is taken, the obliquity of the incident ray is always increased by this angle of incidence H A P.

16. Therefore in great angles such as from 115 to 120 degrees, the increased angle of incidence is so great, and falls so oblique on the index glass and passing through so much of the glass, that the image becomes so faint, as to admit of a secondary image reflected from the first or unsilvered surface of the index glass; and may sometimes occasion considerable errors: but in such cases it is best to measure the angle at twice; as directed in the foregoing part of this work, section 3, in the use of the sextant, fig. 5.

17. But observe, what I have said here concerning the secondary image, is with
regard

regard to terrestrial objects only: for in observations of the sun or moon, there will always be a complication of images; owing to the many refractions and reflections of such bright objects in passing from the different surfaces of the glasses.

18. Care should be taken that the glasses are placed perpendicular to the plane of the instrument; and to know this, set the index near 60 on the limb, then look in the index glass for the image of the limb of the sextant; and observe, if both the image and limb coincide, this glass is perpendicular; otherwise not.

19. The horizon glass may be proved every observation thus: hold the sextant in an horizontal position, and look at any small distant object or a white set up, and see if the image and object appear close together, (the index being at 0) if they do,

this glass is right ; but if the image appear in a perpendicular line at a considerable distance, either under or over the object ; this glass is not perpendicular to the plane of the instrument, and therefore should be corrected.

20. I have taken no notice of the horizon glass, for what is called the back observation ; as it is of no use to the surveyor ; nor would it be of any, I think, to the navigator, if my new method of finding the longitude at sea, was made public.

SECTION

SECTION II.

Of the use of the sextant in taking of heights and distances, by a new method, independent of trigonometry.

21. **T**H E height or distance of any object may be found very expeditiously and exact; by help of the sextant, joined with the following small table of angles.

A TABLE for *Heights and Distances.*

Mul.	Angle.	Angle.	Div.
1	45°,,00'	45°,,00'	1
2	63 „ 26	26 „ 34	2
3	71 „ 34	18 „ 26	3
4	75 „ 58	14 „ 02	4
5	78 „ 41	11 „ 19	5
6	80 „ 32	9 „ 28	6
8	82 „ 52	7 „ 08	8
10	84 „ 17	5 „ 43	10

P R O B L E M I.

22. *To find the height of any accessible object as a tower, chimney, tree, &c. the ground being nearly level.*

R U L E.

Make a mark with chalk, or stick up a bit of white paper, equal the height of the eye from the ground, upon the object. Then set the index of the sextant to any of the angles given in the above table; and walk from the said object, till the image of its top appears at, or coincides with the chalk mark; then if the angle made use of, be greater than 45 degrees; multiply your distance from the object by the figure in the first column standing against the angle made

SEC. II. HEIGHTS AND DISTANCES. 359

made use of: but if the angle used be less than 45 degrees; divide your distance by the figure in the last column standing against the angle made use of. And the product or quotient will be the height of the object above the mark, accordingly; to which add the height of the mark (or of your eye), and the sum will be the whole height of the object from the ground.

23. If the ground be unlevel, the mark may be fixed by help of the spirit tube, as directed in the 3d section of the foregoing part of this work.

24. EXAMPLE. To find the height of the tower A B, (fig. 4.) Having fixed the mark E, first set the index of the sextant to $45^{\circ}, 00'$, then walk back from the tower, suppose to the point C, till the image of its top coincides with the mark

A a 4

E the

E, (the sextant being held in a vertical position), then measure the distance C E, or its equal D A, suppose 30 yards; and this will be equal the height of the tower above the mark E; for the figure in the table against the angle 45° is 1, and therefore the distance D A, multiplied or divided by 1, makes no alteration.

Secondly, if the index be set to $63^{\circ}, 26'$, and you walk back, suppose to F, when the image of its top is brought down to E, then measure the distance F E or its equal G A, and it will be found 15 yards, and this multiplied by 2, (the figure in the first column against $63^{\circ}, 26'$) gives 30 yards or 90 feet as before, for the height B E; to which add A E, suppose 5 feet, and the sum will be the whole height of the tower, equal 95 feet.

25. *Note.* The sextant must be adjusted by article 13, (if necessary) in the above and all such cases, to the distance D A, or G A.

P R O B L E M II.

26. *To measure the height of any inaccessible object, and also to find its distance at the same time.*

R U L E.

Set the index of the sextant to the greatest angle in the table, that the least distance from the object will admit of, when (by moving a little backwards or forwards) the image of the top of the object is brought to a level with the eye; and at the place where this happens, set
up

up a white equal to the height of the eye. Then set the index of the sextant to any one of the other lesser angles in the table, and go back in a direct line from the object, till its top is depressed to a level with the eye, or to the white before set up; and here set a mark also; measure the distance between these two marks, and this distance divided by the difference of the figures in the last column standing against each angle made use of, the quotient will give the height of the object above the height of the eye,

27. *And for the distance.*

Multiply the height of the object, by the figure standing against either of the angles made use of, and the product will be the distance of the object from the place where such angle was used.

EXAM-

EXAMPLE I.

28. Suppose A B (fig. 5.) be a section of the wall of a fort, which is surrounded by a ditch, whose unknown breadth is A C ; and it is required to find the height of the wall, the breadth of the ditch, and the length of a ladder that will reach from the edge of the ditch to the top of the wall.

First take the altitude of the wall at C, suppose $62^{\circ}, 40'$, this shews that you cannot make use of a greater angle, therefore set the index of the sextant to the next less in the table, viz. $45^{\circ}, 00'$, and go back till the image of B is depressed to E (in a level with the eye) suppose to the point 1, where set up a white. Secondly, set the index to any one of the lesser angles, suppose

pose $26^{\circ}, 34'$, and go back in a direct line till the image of B is again depressed to E, or to the white at the point 1; which suppose at the point 2: measure the distance between the points 2 and 1, suppose 34,8 feet, and this divided by 1, (being the difference of the numbers against $45^{\circ}, 00'$, and $26^{\circ}, 34'$) gives 34,8 feet for the height of the wall; and this height multiplied by 1, the figure against the angle $45^{\circ}, 00'$, gives 34,8 feet for the distance of the wall from the point 1; or the same multiplied by 2, the figure against the angle $26^{\circ}, 34'$, gives 69,6 feet, the distance from the point 2; then measuring the distance C 1, suppose 16,8 feet, this taken from A 1, leaves A C, the breadth of the ditch, 18 feet; and the length of the ladder C B is had (by the 47th Euc. 1.) equal 39,2 feet.

EXAM.

E X A M P L E II.

29. Suppose you could get no nearer the fort A B (fig. 5.) than the point D, to find the height and distance.

The greatest angle (correspondent to one in the table) that can be taken, is $26^{\circ}, 34'$, and the object hath such altitude at the point 2. Set the index to $14^{\circ}, 02'$, and the object's altitude will be such at the point 4; measure the distance 4, 2, and it will be found 69,6 feet; which divided by 2, (the dif. for angles) gives 34,8 feet for the height; and the distance from the point 2, is equal the height multiplied by 2, equal 69,6 feet.

30. All which plainly appears from fig. 5, in which 1, 2, 3, 4, &c. may be supposed to represent the figures in the right hand

hand column in the table, (for heights and distances) and their distance each equal the height EB ; then the angles $E_1 B$, $E_2 B$, $E_3 B$, &c. are each equal their correspondent angles; the same as in the table.

E X A M P L E III.

31. What is the perpendicular height of an hill whose angle of elevation at C , near the bottom (see fig. 6.) was 45° , and in a direct line and level with the bottom of it, at D the angle was $26^\circ, 34'$.

Measure DC , 98,5 yards, and that will be equal CA , equal AB , the height of the hill required.

32. Or if the angle ACB had been $26^\circ, 34'$, and the angle ADB $18^\circ, 26'$ (see the table) the distance DC would be the same, equal the height AB .

33. The

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33. The same would be produced with the angles $18^{\circ}, 26'$, and $14^{\circ}, 02'$, &c.

E X A M P L E IV.

34. Being on an horizontal plane, and wanting to know the height of an object on the top of an inaccessible hill: I took the top of the hill's elevation 45° at C, (fig. 7.) and the top of the object's elevation the same at D; then D C measured is 35.5 yards, the height of the object B C.

35. Or the height of the hill, and then the height of the hill and object together may be found by (31), and the hill taken from the whole, leaves the height of the object B E.

E X A M P L E V.

36. Observing the shade of a cloud pass by me, and at $1', 05''$ after, its shade fell on
a spot

a spot at the distance of 250 yards from me on a level plane ; the cloud having passed over me ; I observed, when its altitude was 45° ; and $4', 20''$, after I observed its altitude $26^{\circ}, 34'$; hence the height of the cloud, and its distance at each observation is required.

Suppose A (fig. 8.) the place of observation, and C and D the cloud's situation at each respective observation ; then say, as $1', 05'' : 250 \text{ yards}, :: 4', 20'' : 1000 \text{ yards}$ the cloud's motion in that time, equal C D, equal A B the perpendicular height of the cloud. Hence the distance A C or A D becomes known.

PROBLEM

P R O B L E M III.

37. *To find the distance of any inaccessible object from a given base line.*

R U L E I.

Having chosen your first station, stick up a white; also at a proper distance, and at right angles to the object whose distance is wanted, set up another white; then set the index of the sextant to any of the angles given in the table, greater than 45° ; and go back, keeping the whites in a right line, till the said whites coincide with the object, (or the object coincides with the whites) at which place, set up a mark and call it your second station: measure the distance between the stations,

B b

and

and multiply it by the figure in the first column standing against the angle made use of; and the product will give the distance of the object from the first station.

R U L E II.

38. If the object be so situated that you cannot obtain a right angle or 90° , at the first station, with the base line; then use any one greater than 45° , that is in the table; by help of which, set up the mark at a distance. Set the index to any lesser angle given in the second column of the table; go back, keeping the whites in a right line, till they coincide with the object; and there mark your second station; then multiply the distance between the stations, by the figure in the first column against the angle at the second station, divided

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divided by the difference of the figures against each angle; and the product gives the distance of the second station from the perpendicular line, let fall from the object to the base line; and this distance multiplied by the said figure against the second angle, will give the perpendicular distance of the object from the base line.

39. Or the distance of the stations, being multiplied by the product of the two numbers (against the angles made use of) divided by their difference, will give the perpendicular distance of the object from the base line.

40. It may be more convenient sometimes to use the lesser angle first, and then go towards the perpendicular; in this case, the foregoing rule (38.) gives the distance of the perpendicular from the first station: but the rule is general, always

giving the distance of the perpendicular from the station farthest from it.

41. But if particular angles (from the table) are made use of, as may in general be the case; a shorter rule may be deduced; thus, if the angles against the multipliers 1 and 2, or 2 and 4, or 3 and 6, or 4 and 8, or 5 and 10, are used for the two stations; then the distance of the nearest station to the perpendicular, will be equal to the distance between the two stations; and this distance multiplied by the figure for the nearest station, will give the perpendicular distance of the object from the base line.

EXAMPLE I.

42. A tree is observed to grow close to the further side of a river as at B, (fig. 9.)
required

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required the distance A B of the tree, or breadth of the river.

Suppose A D a base line on the near side of the river ; find the point A where a perpendicular from B falls on this line ; find an angle A C B (equal one in tab.) suppose $75^{\circ}, 58'$, then measure C A, suppose 20 yards, and this multiplied by 4, (the multiplier against $75^{\circ}, 58'$) gives 80 yards for the distance A B, or breadth of the river.

E X A M P L E II.

43. Wanted the length A B (fig. 10.) of a pool, part of whose sides could not be measured, by reason of brush wood, &c.

At the end A erect the perpendicular base A D (by help of the sextant), find an

B b 3 angle

angle A C B, $78^{\circ}, 41'$, measure C A 2,24 chains; and this multiplied by 5, (see table) gives 11,20 chains for the length of the pool A B.

E X A M P L E III.

44. A ship sailing away due west, sees a point of land nearly N. N. W. viz. $63^{\circ}, 26'$ from the west; and after sailing 4 leagues or 12 miles, the land bears due N. required the ship's distance from the land at each observation.

Suppose B (fig. 11.) the point of land, C the place of the ship at first, and A at second observation; then the angle A C B is $63^{\circ}, 26'$, and C A 12 miles, which multiplied by 2 gives A B, 24 miles, the ship's distance, at the second observation: hence B C the distance at the first observation, is

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is had (by 47 EUCLID. 1.) equal 26,8 miles.

$$\text{For } \sqrt{A B^2 + A C^2} = BC = 26,8.$$

E X A M P L E IV.

45. Some ships of war intend cannonading a fort B (fig. 11.) but by the shallowness of the water, are kept so far from it, that they suspect their guns cannot reach it; in order therefore to measure the distance, one of them separateth, and sailing towards D, (making the angle B C D $97^{\circ}, 08'$) until the angle B D C is $71^{\circ}, 34'$, and found C D, his distance sailed, to be just half a mile.

$$\begin{array}{rcl} & 180^{\circ}, 00' \\ & 97^{\circ}, 08' \\ \text{hence is given} & \left| \begin{array}{l} \text{Mul.} \\ 8 \\ 3 \end{array} \right| \overline{\begin{array}{l} 82^{\circ}, 52' \\ 71^{\circ}, 34' \end{array}} & \left. \begin{array}{l} \angle ACB \\ \angle ADB \end{array} \right\} \& C^D = 0,5 \\ & B b 4 & \text{then} \end{array}$$

then (38.) $0,5 \times \frac{8}{8-3} = 0,8 = AD$, and
 $0,8 \times 3 = AB = 2,4$ miles; also $AD -$
 $CD = AC = 0,3$; and $\sqrt{AC^2 + AB^2}$
 $= BC = 2,58$ miles the distance from the
 fort required.

EXAMPLE V.

46. A surveyor taking the map of a river, sees at a distance on the other side of the river, a church B (fig. 12.) and a windmill E; whose distance from the river, and also from each other he wanted to know.

At A he found the angle CAB 90° , and CAE $75^\circ, 58'$; at C, the angle ACB $78^\circ, 41'$, DCE $82^\circ, 52'$; and the measured distance AC 300 yards.

First

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First then (37.) $A C = 300 \times 5 = A B = 1500$. Secondly (41.) $A C = 300 = C D$, and $C D = 300 \times 8 = E D = 2400$. and $B F = A D = 600$, also $E F = E D - A B = 900$; then $\sqrt{B F^2 + E F^2} = B E = 1081,9$ yards.

47. If $A B E C$ (fig. 12.) represent the sides of an inclosure, whose content and plot may be wanted; but the surveyor not permitted to go nearer than a publick road joining the side $A C$. It may easily and exactly be obtained by the foregoing rules.

SECTION

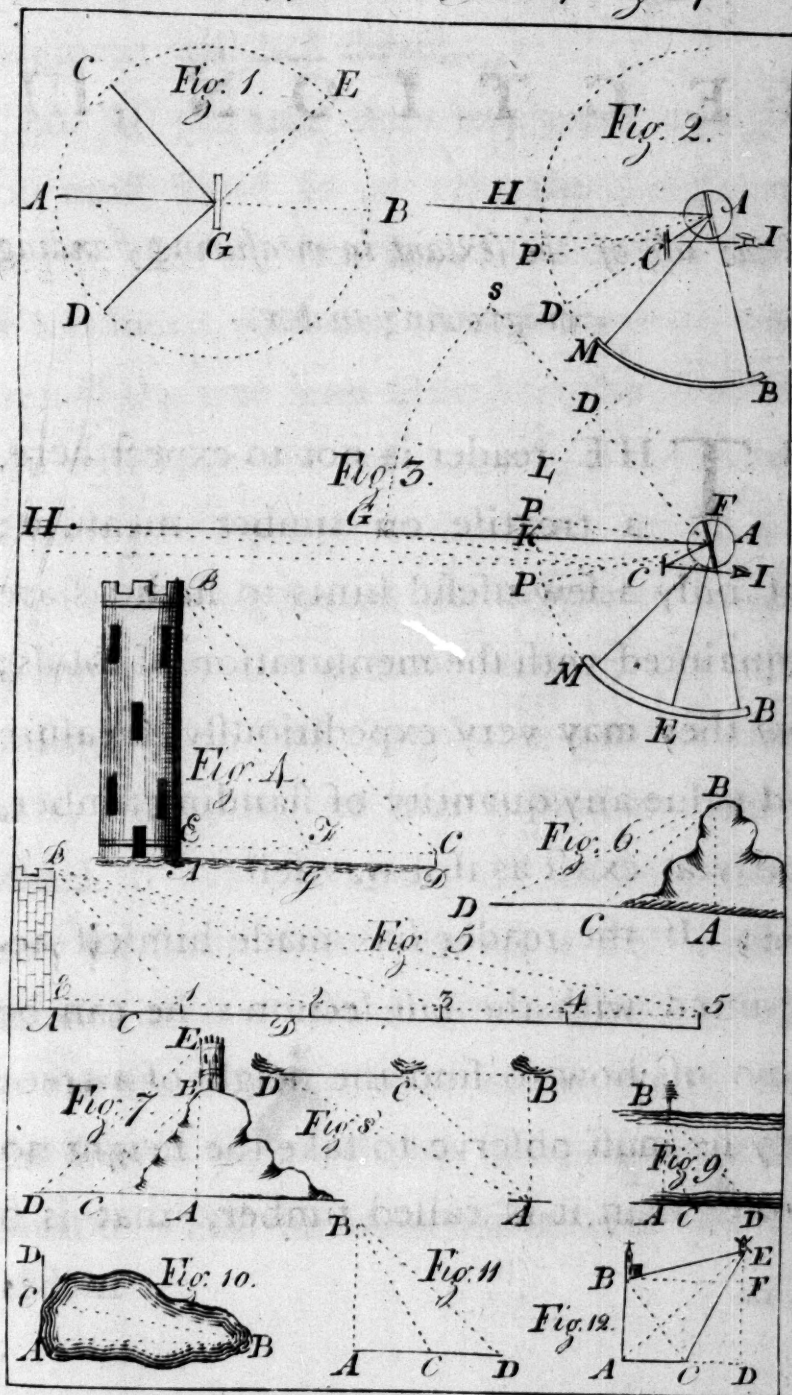
S E C T I O N III.

Of the use of the sextant in measuring standing or growing timber.

48. **T**HE reader is not to expect here, a treatise on timber measure; but only a few useful hints to such as are acquainted with the mensuration of solids; how they may very expeditiously measure and value any quantity of standing timber, nearly as exact as if it was fell.

49. If the reader has made himself acquainted with the last section; he can be at no loss how to find the height of a tree: only he must observe to take the height no farther than it is called timber, that is 6
inches

Plate Appendix to face page 378



inches girt or quarter compass, or 24 inches in circumference ; which his own judgment will best direct.

50. If the tree does not grow upright you must stand so to take the height or length, that the tree may lean to the right or left hand, and not to or from you ; and then if the tree lean almost to the ground, you will have the true length, if the image of its top be brought to the chalk mark, &c.

51. Having found the height or length of the tree so far as it is timber ; the next business is to find the girt, or quarter compass, according to the customary way ; which if the tree grow tall and tapering, may be found thus : measure the circumference of the tree near the ground, viz. at the spreading abroad of the roots, and the fourth part of this is the girt at the bottom ; and six inches is always (in this case)

case) the girt at the top ; and half the sum of the top and bottom girts, gives the girt in the middle as required.

52. But in pollards or trees that have had their tops cut off, and their branches frequently lopped for fuel, as is the custom in some countries ; you may commonly by small discretionary allowance, make the mean or middle girt a little less than it is, five or six feet from the ground : for some of those trees being almost as thick at the top as the bottom, therefore by a little care and practice the difference may be easily accounted for.

53. However, as the foregoing methods of finding some of the dimensions, may be thought by some persons, too imperfect, especially in very large and tall timber : I shall therefore proceed to shew how to
take

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take all the dimensions as exact as if the trees were fell.

54. If the tree, whose content you want, is tapering to the top, it is necessary to find the place where it is only 6 inches girt, or 24 inches in circumference; and may be very easily found thus; having made a chalk mark equal to the height of your eye from the bottom of the tree, and parallel to the ground or horizon, cross it with two perpendicular short lines at the distance of 5,4 inches from each other; then setting the index of the sextant to 45 degrees on the limb, go back from the tree till the breadth of the image of the top part of the butt of the tree be just equal to the space between the two perpendicular chalk lines, viz. 5,4 inches: then will your distance from the tree, be just equal to

to its height or length, so far as it is called timber, above your chalk mark.

55. Take the girt (that is the fourth part of the circumference) near the bottom, to which add 6 inches the girt at the top; half this sum is the girt in the middle, therefore having obtained the length and girt in the middle, the content is found as usual.

56. If any obstacle should prevent your going so far back from the tree, as the angle 45 degrees requires; then use the angle $63^{\circ}, 26'$, and go back till the breadth of the image of the top part of the tree is but 3, 4 inches; then will the tree's height or length, be just double (22.) your distance from the said tree.

57. If the top of the tree's butt, where the branches issue or spread out from the said butt, exceed six inches girt; then
having

SEC. III. STANDING TIMBER. 383

having set the index of the sextant to 45 degrees, go back till the image of such part of the tree where the branches do thus begin to spread, is brought to the chalk mark; and there set down a staff or other mark: set the index of the sextant to the next angle in the table, less than 45 degrees, viz. $26^{\circ}, 34'$, and looking at the chalk mark, observe what part of the tree is seen in the quick-silvered part of the horizon glass to coincide with the said chalk mark; which will be the middle of the tree, or of that part above the chalk mark.

58. Then take the angle of the tree's diameter in such place (or middle just found) by bringing the image of the right hand side of the tree, to coincide with the left hand side; which angle enter in your wood-book; take the angle also of the tree's

tree's diameter at the chalk mark, which also enter ; then measuring your distance from the tree, gives the height or length.

59. Measure the tree's girt (that is, the fourth part of the circumference) at the chalk mark, and say by the rule of proportion, as the angle at the bottom (or chalk mark) is to it's girt, so is the angle at the middle to it's girt. But as your distance from the middle is greater than from the bottom, this fourth term must be increased in the same ratio ; this may always be done (when the angle 45 degrees is made use of to find the tree's height) by multiplying the fourth term by 1,1 : all which is performed by the sliding rule with the utmost ease and sufficient exactness.

60. The middle girt here found, is the middle between the chalk mark and the top ; and consequently the content of that
part

SEC. III. STANDING TIMBER. 385

part must be found, and added to the content of the part below the chalk, which is easily measured.

But in practice, it will be better to take the angle about $1\frac{1}{2}$ or two feet below the place indicated by the chalk mark, according as the part between the said mark and the spreading of the roots at the bottom is 3 or 4 feet, &c. and the girt found to this angle will be sufficiently near the middle, between the top and bottom of the tree; and then for the length, add the part between the chalk mark and the spreading of the roots, to the distance or height before found, the sum will be the whole length.

61. If the angle $63^{\circ}, 26'$ be used for the distance, the part below the chalk mark must be added to twice the distance for the whole length; and the girt found in the

C c

middle

middle by proportion, must be increased in the ratio of 1 to 1,4: that is, multiply the girt found by proportion, by 1,4, and the product will be the true girt in the middle. A few examples will make all plain to the meanest capacity.

E X A M P L E I.

When a tree grows tapering to less than six inches girt.

Having made a mark on the tree 5 feet from the ground with chalk, parallel to the horizon, and crossed it with two short perpendicular lines at the distance of 5,4 inches; I set the index of the sextant to 45 degrees, and went back from the tree, till the image of the top part of the tree was equal in breadth to the space between the chalk lines or 5,4 inches; I then measured

SEC. III. STANDING TIMBER. 387

fured my distance from the tree, and found it 26 feet; I also measured the circumference at the spreading of the roots, viz. 4 feet below the chalk mark, and found it 10 feet, the fourth of which is 2 feet 6 inches for the girt at the bottom, to which add 6 inches for the girt at the top, and the sum is 3 feet, the half of which, 1 foot 6 inches, is the girt in the middle. And the whole length is 4 feet added to 26 feet the distance, equal to 30 feet.

Then 1,5 the girt
 1,5 the same

2,25 square of girt
30 length

Feet 67,50 the content = $67\frac{1}{2}$ ft.

By the SLIDING RULE.

In. Ft. In: Ft.

As 12 : 30 : : 18 : $67\frac{1}{2}$ content.

That is, set 12 on the girt line to 30 the length in feet on the slider, and against 18

C c 2

inches

inches on the girt line (being the middle girt) is $67\frac{1}{2}$ feet, the answer on the slider.

62. It is easy to observe, that if the middle girt of any tree be just one foot; then the content will be just equal the length in feet and inches of such tree.

Hence is deduced another easy solution by the sliding rule, viz. Set one foot on the girt line to 30 feet, the length on the slider, and against 1,5 feet the middle girt, is $67\frac{1}{2}$ feet the content as above.

And this rule is general, when the dimensions are taken in feet and decimal parts, or reduced to such.

E X A M P L E II.

63. When the top of a tree, at the spreading of the branches, girts more than 6 inches.

Having

SEC. III. STANDING TIMBER. 389

Having made a chalk mark 5 feet from the ground, I set the index to 45 degrees, and went back from the tree till the image of the top of the butt (or the beginning of the spreading of the branches) was brought down to the chalk mark, and set down a staff. Then I set the index to $26^{\circ}, 34'$, and observe what part of the tree's butt has it's image coinciding with the chalk mark; and at about 2 feet below such part, I took the angle of its diameter, and found it $3^{\circ}, 30'$; I also took the angle of diameter at the chalk, and found it $5^{\circ}, 30'$. I then measured my distance from the tree, and found it 24 feet; I also measured the girt at the chalk, and found it 1,81 feet, all which I entered in the wood-book thus, each under its proper title; see the wood-book in the specimen following. Then I say by the sliding rule

C c 3

on

on the lines of numbers, as $5,5^{\circ} : 3,5^{\circ} ::$
 $1,81 \text{ ft.} : 1,15 \text{ ft.}$ this multiplied by $1,1$,
 gives $1,26$ feet for the middle girt : or by
 the sliding rule thus ; as $1 : 1,1 :: 1,15$
 $\text{ft.} : 1,26 \text{ feet.}$ Then for the content by
 the girt line on the sliding rule, (62.) as
 $1 : 28 :: 1,26 : 44,5$ feet the content.

The same by the PEN.

1,26 middle girt
62,1 same inverted
126
25
8
8,59 square of girt
28 length
1272
318
44,52 feet. Content,

EXAM-

E X A M P L E III.

64. When an obstruction prevented my using the angle 45 degrees.

I set the index to $63^{\circ}, 26'$, and went back till the tree's top was depressed to the chalk; then I set the index to 45 degrees, and observed the middle; and at $2\frac{1}{2}$ feet below, I took the angle of diameter $7^{\circ}, 35'$; I also took the angle at the chalk $13^{\circ}, 25'$, and measured my distance $10\frac{1}{2}$ feet, which doubled, and the 5 feet below the chalk added thereto, gives 26 feet for the whole length of the tree. I then measured the bottom girt 1,96 feet, and say, as $13^{\circ}, 4 : 7^{\circ}, 6 :: 1,96 : 1,11$ feet; this multiplied by 1,4 gives 1,55 feet for the middle girt.

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Then for the content by the fliding rule,
(62.) as 1 ft. : 26 ft. :: 1,55 : 62,7 feet,
the content. Or thus by the pen ;

1,55 middle girt
55,1 inverted

155
78
8

2,41 square girt
26 length

1446
482

62,66 feet. Content,

65. Here follows a specimen of a Wood-
book, containing the foregoing examples :

No. trees.	Stationary angle.		Bottom angle.		Middle angle.		Whole length.	Bottom girt.	Middle girt.	Content.
	D.	M.	D.	M.	D.	M.	F.	F.	F.	F.
1	45	00	-	-	-	-	30	2,5	1,5	67,5
2	45	00	5	30	3	30	28	1,81	1,26	45,5
3	63	26	13	25	7	53	26	1,96	1,55	62,7

SECTION IV.

The use of the Sliding Rule in casting up or finding the contents of land.

66. **T**O find the content of a triangle by having the base and perpendicular given.

R U L E.

Say (by the lines of numbers A and B) as 20 on the line A is to the base in chains and decimals on the line B; so is the perpendicular in chains, &c. on A, to the content in acres and decimals on the line B.

EXAM-

E X A M P L E.

67. Given the base 12,60 chains and the perpendicular 7,25 chains to find the content.

As 20 : 12,6 :: 7,25 : 4,57, that is, set 20 on A, to 12,6 on B, and against 7,25 on A is 4,57, the content on B, = 4 acres, 2 roods, 11 perches.

68. *To find the area of a triangle by having two sides, and the included angle given.*

R U L E.

First, set 10 on A, to the length of one side on B, and against the length of the other side on A, is a fourth term, on B. Secondly, set 1 on A, to this fourth term on B, and against the factor for the given angle

angle on A, is the answer or content on B, in acres and decimal parts.

E X A M P L E.

69. Given one side 8,50 chains, and the other 9,20 chains, and the included angle 80 degrees, whose factor (by Tab. 3.) is ,492.

Then as $10 : 8,5 :: 9,2 : 7,8$, fourth term ; And as $1 : 7,8 :: ,492 : 3,85$, the content equal 3 acres, 3 roods, 16 perches

70. *Having given the diagonal and sum of the perpendiculars of any trapezium, to find the content by the sliding rule.*

R U L E.

Set 20 on A, to the diagonal on B, and against the sum of the perpendiculars on A, is the content on B.

EXAM-

E X A M P L E.

70. Given the diagonal 12,42 chains, and the sum of the perpendiculars 13,25 chains; as $20 : 12,42 :: 13,25 : 8,23$ acres, equal 8 acres, 0 roods, 37 perches.

71. *To find the area of a trapezium by having the sides and angles given.*

R U L E.

Suppose a diagonal to be drawn, and it reduces it into two triangles, which cast up by (68) and their sum will be the content of the trapezium.

E X A M P L E.

72. Given the sides of one part 6,8 and 6 chains, and the included angle $92^{\circ}, 45'$,
whose

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whose factor is ,499. And the other part 6,20 and 4,8 chains, and the included angle 115 degrees, whose factor is ,453.

Then as 10 : 6,8 :: 6 : 4,08.

And as, 1 : 4,08 :: ,499 : 2,04 acres.

Again, as 10 : 6,2 :: 4,8 : 2,98.

And as 1 : 2,98 :: ,453 : 1,35 acres ;
and their sum is 3,39 acres, equal to 3
acres, 1 rood, and 22 perches.

73. *To reduce statute measure to customary.*

R U L E.

Say, as the square of the customary perch is to the given number of statute acres, so is the square of the statute perch, to the number of customary acres.

E X A M P L E.

74. How many customary (*Lancashire*)
acres of 8 yards to the perch, are con-
tained

tained in 42,3 statute acres of $5\frac{1}{2}$ yards to the perch.

As $64 : 42,3 :: 30,25 : 20$ the number of customary acres.

75. As the valuation of land, is of as much importance as measuring; I have added the following table, shewing the value from one to fifty shillings an acre, of any quantity of land.

76. *Explanation and use of the TABLE.*—The first column contains the value, per acre; the second, the value of three roods; the third, of two roods, &c. (See the table.)

77. *To find the value of any quantity of land, at any rate per acre, by the table.*

R U L E.

Multiply the number of acres by the rate or value per acre, to which add from the

the table, the numbers for the roods and perches standing against such value, and their sum will be the true value.

E X A M P L E.

78. What is the value of 12 acres, 3 roods, 20 perches, at 7s. an acre.

	£.	s.	d.
Then 12 acres × 7s.	=	4	04 00
3 roods	=	0	05 3 per table.
20 perches	=	0	00 10½ ditto.
Answer	=	4	10 1½

E X A M P L E II.

79. What is the value of 8 acres, 3 roods, and 23 perches, at 12 shillings an acre?

	£.	s.	d.
8 acres, at { 10 is	=	4	00 : 0
2 is	=	0	16 : 0
3 roods, at { 10 is	=	0	07 : 6
2 is	=	0	01 : 6
20 pch. at { 10 is	=	0	01 : 3
2 is	=	0	00 : 3
3 pch. at { 10 is	=	0	00 : 2½
2 is	=	0	00 : 0½
Answer.	=	5	06 : 8½

EXAMPLE

EXAMPLE III.

80. What is the value of 30 acres, 2 roods, 16 perches, at 3s. and 6d. an acre.

		£.	s.	d.
Here 30 acres, at	{ 3s. is	=	4	: 10 : 0
	{ 6d. is	=	0	: 15 : 0
2 roods, at	{ 3s. is	=	0	: 01 : 6
	{ 6d. is	=	0	: 00 : 3
10 pch. at	{ 3s. is	=	0	: 00 : 2 $\frac{1}{4}$
	{ 6d. is	=	0	: 00 : 0 $\frac{1}{4}$
6 pch. at	{ 3s. is	=	0	: 00 : 1 $\frac{1}{2}$
	{ 6d. is	=	0	: 00 : 0 $\frac{1}{4}$
Answer.		=	5	: 07 : 1 $\frac{1}{4}$

Note. For the value at 6d. take half the value at 1s &c.

81. The second and third examples, as well as many others, may be abbreviated, by finding the value to half the price, and then doubling it; or finding the value of double the price, and taking the half thereof; will give the true value in either case.

Thus

Thus in EXAMPLE II.

	£.	s.	d.
The value of 8 acres, at 6s. is	2	8	0
3 roods, at do. is	0	4	6
20 perch. at do. is	0	0	9
3 perch. at do. is	0	0	1 $\frac{1}{4}$

value at half the price 2 13 4 $\frac{1}{4}$

Add the same 2 13 4 $\frac{1}{4}$

the sum is the true value as before 5 6 8 $\frac{1}{2}$

Also in the third EXAMPLE.

	£.	s.	d.
The value of 30 acres, at 7s. is	10	10	0
2 roods, at do. is	0	3	6
10 perch. at do. is	0	0	5 $\frac{1}{4}$
6 perch at do. is	0	0	3

The value at double the price 10 14 2 $\frac{1}{2}$

The half gives the true value = 5 7 1 $\frac{1}{2}$

82. If land has coal or other mines in it, and is let or sold by the acre; the price or value per acre, may much exceed the limits of the foregoing table; yet notwithstanding this, its value may be found very easily by help of the said table, as follows :

D d

First

402 VALUATION OF LAND. (APPEN.)

First find the value of the given quantity of land, at 20 shillings or one pound an acre by the table; then multiply this value by the value of the land per acre; and the product will be the answer.

E X A M P L E.

Suppose 60 acres, 2 roods, and 3 perch. having a mine of coal in it, which is sold for 30 pounds an acre; What is its value?

	£.	s.	d.
First the value of 60 acres, 2 roods, 03 perches, at 20 s. or 1 l. per acre, is worth by the table	-	-	-
Multiply by	60	10	4½
the value at 6 l. an acre	363	2	3
Multiply by			5
Value at 30 l. an acre is	1815	11	3

In the same manner may the value of an estate be found, at any number of years purchase.

SECTION

S E C T I O N V.

Of MEASURING of ROADS.

83. **I** Intended in the foregoing part of this work, to have added *County and Coast Surveying*; but found so many new and useful improvements, which are absolutely necessary in making a compleat and accurate survey of a county or a coast; that I found it would swell this volume too much to come within due bounds, as well as *inhance* the price too much. But if what I have already done, meets with a favourable reception by my friends and the public in general; It may, perhaps, induce me at some future opportunity, to

lay before them the whole of my improvements in *County Surveying*.

84. In measuring of roads, there are chiefly two instruments in use, viz. the perambulator, way-wiser, or surveying wheel, and the chain; the former has generally been made use of, as being more easy and more expeditious than by the latter; but where great exactness is required, I would recommend the use of the chain; for I have often observed by following a person driving the surveying wheel, as also when I have driven it myself with the utmost care, that its track was so irregular and waving to the right and left, as well as passing over stones and into holes, that in measuring a mile or two, it would encrease the distance very considerably, and much more so in great distances; but the
chain

chain is not subject to any such errors. —I shall therefore give the young surveyor a few directions for measuring roads by the chain only ; and shew him how to keep a road-book.

85. If the road to be surveyed lead from a town, then begin at some remarkable place in the town, as the market-cross (if there be any), or church, &c. which place, insert at the top of your road-book, which having three columns near the middle, marked *M. F.* and *P.* to insert the miles, furlongs, and poles ; and the part to the right and left, are for inserting remarks, &c. Having inserted the place where you begin, let your assistant lead the chain and keep along the middle of the road as near as he can, measuring on till you come to some remarkable object, as a cross street, end of the town, cross lane, &c.

D d 3

which

which enter in the column of remarks, and if it be a cross road enter where it comes from, and leads to, &c. drawing two short parallel lines to represent the road, observing to enter the objects that are on the right side of the road, in the right hand column of remarks, and objects to the left hand, in the left hand column, &c. and when you have measured ten chains, enter 1 in the column marked *F*, for one furlong; and at the next ten chains, enter 2 furlongs, &c. to 8 furlongs or 80 chains; for which enter 1 in the column of miles, and 0 in the column of furlongs, and when there are any odd chains or chains and links, at any remark, over and above the furlongs, or miles and furlongs; multiply them by 4, and set the product in the column under *P* or poles; for one chain makes four poles, and

and ten chains makes forty poles or one furlong ; therefore any number of chains less than ten, being multiplied by 4, the product will be poles.

86. But instead of writing the changes or furlongs, regularly one under another in the column of furlongs, it will be better to enter them in a line in the right hand column of remarks ; writing only the furlongs and poles in the middle columns that occur against their respective remarks ; and when you have measured eight changes or one mile, draw a line across your road-book, and enter one in the column of miles just under the line, and cyphers in the columns of furlongs and poles. The following specimen of a road-book will make the whole plain to the meanest capacity.

*A Survey of the Road leading from W—LL
CROSS to C—H BRIDGE, in the Street-
way road, in the Parish of C—K; taken
December, 1764, by B. T.*

R O A D B O O K.

Remarks, &c.	M	F	P	Remarks & Changes.
W—l Crofs	0	0	0	1 2 3 4 5 6 7 8
Hall-lane		1	10	
		1	24	the bridge
Road to W. Hamp.=		2	36	
		3	20	end of town
Blue lane =		5	8	
	1	0	0	1 2 3 4 5 6 7 8
		2	20	Hollybush stile
		5	12	Broad stone
To Bentley =		7	30	= to Harding
	2	0	0	1 2 3 4 5 6 7 8
Mopes Ash		0	20	to Blakenol
to Bentley =		1	12	
		2	36	Bloxwich hall
to Effington =		4	36	
Bloxwich Chapel		5	4	
enter Shortheath		6	36	

R O A D B O O K continued.

Remarks, &c.	M	F	P	Remarks & Changes.
Turnpike from } W. Hampton }	3	0	0	1 2 3 4 5 6 7 8 = to Lichfield } off Shortheath }
	2	30		= to little Wirley
	6	36		Ellsfield Hall
enter Effington Com.	4	0	0	1 2 3 4 5 6 7 8 end W — ll Parish
a House enter C — k Parish	5	14		a small brook off Effington Com.
	5	18		
first Ale-house in	5	0	0	1 2 3 4 5 6 7 8 = to Little Wirley
	2	12		Great Wirley
	7	8		
Old Loggerheads	6	0	0	1 2 3 4 5 6
C — h br. street road	4	16		
	6	28		= London road

Note. If the road is to be planned, then take the most material angles as you go along, and place them to the right or left column, as they happen to be observed.

REMARK.

R E M A R K.

87. **S**INCE I wrote the 12th Section on the Dividing of Commons, &c. I have made enquiry into the methods used by the best practical surveyors, and find that they measure the land of different value in separate parcels, and find the separate values thereof, which added into different sums, gives the whole content, and also the whole value. Then (by Prob. 1. Sect, 12.) they find each man's proportional share of the whole value; and then lay out each person a quantity of land equal in value to his share. This they do by first laying out a quantity by guess, and then casting it up and finding its value; and if such value be equal to his share of the whole value, the dividing line is right; but

but if otherwise, they shift the dividing line a little, till by trial, they find a quantity just equal in value, to the value of the share required.

If any single share contains land of several different values ; each is measured separately, and their several values found ; and if the sum of them be equal the value sought, the division is right ; if not, it must be altered till it does come right.

88. This method is very tedious as well as laborious, but perfectly true both in theory and practice ; and had I known it before I wrote the abovementioned section it would have prevented the insinuations thrown out there as touching the truth of the division of commons.

A CAUTION

A CAUTION to Young SURVEYORS.

89. In measuring single inclosures, it is common to measure to the growers if the ditch be on the outside of the land you are measuring, and to add 5 links for the ditch; then in measuring the next line if there be any off-sets they must be measured only to the hedge, because the chain line is not at the extremity of the ditch, but parallel thereto, and having the hedge and ditch between.



F I N I S.